

Kant and Bolzano on logical form

Paul Rusnock

In *Kant-Studien* 102 (2011) 1–15.

I.

Kant had declared in the *Critique of Pure Reason* that logic (at least the variety he called pure general logic) abstracts from all content (or matter) and attends only to the form of thought. Because it deals with form alone, logic has no empirical principles: every truth of pure logic is *a priori*.¹ This inference is based upon the thesis that everything formal in our cognition is contributed by us. As the Kantian logician Born put it, the matter is in the object, and the form in the subject.² Furthermore, because the form is in us, it would remain the same even were the matter to be radically altered: the formal aspects of cognition are therefore *necessary* in this sense. If now we assume that the understanding is transparent to itself, we can also claim that all of the principles of logic can be known with absolute *certainty*, at least for appropriately trained thinkers, those having, as Kant put it, the “long practice” required to separate the raw material of experience from that which our cognitive faculties add to it.³

This task is rendered all the easier for us because the formal realm is not only open to our gaze, but quite limited in extent and complexity. Contemplating a complete system of the philosophy of pure reason (and logic must surely form part of this), for example, Kant writes:

That such a system is possible—and, indeed, that it cannot be overly wide-ranging, so that we may hope to complete it entirely—can be gathered even in advance from the following: What here constitutes the object is not the nature of things, which is inexhaustible, but the understanding that makes judgments about the nature of things, and even this understanding, again, only in regard to its *a priori* cognition. Moreover, the understanding’s supply of *a priori* cognition cannot be hidden from us, because, after all, we need not search for it outside

¹KrV, A54/B78, A131/B170. English translations from the *Critique of Pure Reason* are from W. Pluhar’s edition (Indianapolis: Hackett, 1996).

²Born, Friedrich Gottlob: *Versuch über die ursprünglichen Grundlagen des menschlichen Denkens*. Leipzig, 1791, 7.

³KrV, B 1-2. Cf. KrV, A xx: “[...] what reason brings forth entirely from itself cannot hide, but is brought to light by reason itself as soon as we have discovered its common principle.” “[...] was Vernunft gänzlich aus sich selbst hervorbringt, sich nicht verstecken kann, sondern selbst durch Vernunft ans Licht gebracht wird, sobald man nur das gemeinschaftliche Princip desselben entdeckt hat.]

the understanding; and we may indeed suppose that supply to be small enough in order for us to record it completely, judge it for its value or lack of value, and make a correct assessment of it.⁴

Given the centrality of the form/matter distinction in their accounts of logic, and the claim that form is both simple and plain to see, one might have expected that the Kantians would have been able to pin it down quite precisely. Yet we are consistently disappointed when we turn to the Kantian logics of the late eighteenth and early nineteenth century, to Kant/Jäsche, Born, Calker, Fries, Kiesewetter, Krug, Maaß, Mehmel, *et al.* Despite all the large claims made on its behalf, there was no generally agreed upon, acceptable definition of logical form, and despite the widespread acceptance of the tables of judgments and categories, there was anything but unanimity in their interpretation and application.

At various places in the *Theory of Science*, Bolzano surveys the remarks of Kant and his followers on form and matter with a critical eye.⁵ Perhaps the most glaring flaw he notes is that no suitable general definition is ever given of “form” and “matter”. We are indeed told what constitutes the form and matter of concepts, various kinds of judgments, and inferences, but there is no indication of why it is appropriate to use the terms “form” and “matter” in all these cases. “Rather, this is simply stipulated, as if it were entirely arbitrary what should be counted as matter and what as form in these objects.”⁶ Moreover, the special definitions are usually vague as well as obscure.

The case of inferences is particularly glaring (which is not meant to imply that the other definitions are any better). In the Jäsche Logic, Kant simply says: “The *matter* of inferences of reason consists in the antecedent propositions or premises, the *form* in the conclusion insofar as it contains the *consequentia*.”⁷ In his Logic,

⁴KrV, B26: “Denn daß dieses möglich sei, ja daß ein solches System von nicht gar großem Umfange sein könne, um zu hoffen, es ganz zu vollenden, läßt sich schon zum voraus daraus ermessen, daß hier nicht die Natur der Dinge, welche unerschöpflich ist, sondern der Verstand, der über die Natur der Dinge urtheilt, und auch dieser wiederum nur in Ansehung seiner Erkenntniß *a priori* den Gegenstand ausmacht, dessen Vorrath, weil wir ihn doch nicht auswärtig suchen dürfen, uns nicht verborgen bleiben kann und allem Vermuthen nach klein genug ist, um vollständig aufgenommen, nach seinem Werthe oder Unwerthe beurtheilt und unter richtige Schätzung gebracht zu werden.” Cf. Kant’s well-known comments on the completed state of logic, KrV, A xiv and B viii-ix. Frege was a faithful Kantian in this respect, as in others: in *The Foundations of Arithmetic* tr. J. L. Austin, 2nd ed. (Oxford, 1980), §5, Frege mentions as one of the requirements of reason that “it must be able to embrace all first principles in a survey.”

⁵See especially §§12, 116, 186ff, 254ff of Bolzano, Bernard: *Wissenschaftslehre*. 4 Vols., Sulzbach, 1837. New edition by J. Berg in the *Bernard Bolzano-Gesamtausgabe* (12 Vols, Stuttgart-Bad Cannstatt: Frommann-Holzboog, 1985-2000), hereafter **WL**. References to this work will follow the pagination of the 1837 edition.

⁶**WL**, §254 [II.515].

⁷“In den Vordersätzen oder Prämissen besteht der *Materie*; und in der Conclusion, so fern sie

Kiesewetter says: “The premises constitute the *matter* of an inference; the way the conclusion is derived from them is called the *form* of the inference.”⁸ Later, he elaborates: “The premises are the matter, and their combination the form of the inference. The form of the inference, the derivation of the conclusion from the premises, is expressed by the word *therefore*.”⁹ Krug has the decidedly unhelpful:

In every inference one must distinguish matter (*materia*) and form (*forma*). The matter consists of the judgments that are combined in the inference, along with the concepts occurring in the latter; the form of the way in which the premises and conclusion are combined so that one determines the other with respect to its validity. Thus in every inference there occurs a manifold that is combined by reason into a unity of consciousness, and thus *thesis*, *antithesis*, and *synthesis*.¹⁰

It is a challenge, to say the least, to figure out what the various possible forms of inference are based upon such definitions. Astonishingly, however, the Kantian logicians arrive infallibly at the same short list.

For a further example of how difficult it is to apply the Kantian definitions, consider the simple question of whether the distinction between analytic and synthetic judgments is one of *form* or *matter*. Kant seems to provide support for each of these apparently incompatible positions. In the Jäsche Logic, he writes that

1. “Extension (b) belongs to every x to which the concept of a body (a+b) belongs” is an example of an analytic proposition.
2. “Attraction (c) belongs to every x to which the concept of body (a+b) belongs” is an example of a synthetic proposition.¹¹

We can almost read off the modern formalizations from Kant’s statements:

1. $\forall x((Ax \wedge Bx) \rightarrow Bx)$

die Konsequenz enthält, die *Form* der Vernunftschlüsse.” *Logik*, §59 [Logik, AA 09.121.12-14] (translation by J. M. Young in: Kant, I.: *Lectures on Logic* [Cambridge: Cambridge University Press, 1992]).

⁸Kiesewetter, Johann Gottfried: *Grundriß einer allgemeinen Logik nach Kantischen Grundsätzen. . . . Begleitet mit einer weitere Auseinandersetzung für diejenigen die keine Vorlesungen darüber hören können*. 4th ed. Leipzig, 1824. Grundriß, 76 (the two parts of the work, though bound together, are paginated separately).

⁹*Ibid.*, Weitere Auseinandersetzung, 254.

¹⁰Krug, Wilhelm Traugott: *Denklehre, oder Logik*. Königsberg. 1806, 285.

¹¹§36 [Logik, AA 09.111.7-11]: “Alles x, welchem der Begriff des Körpers (a+b) zukommt, dem kommt auch die Ausdehnung (b) zu, ist ein Exempel eines analytischen Satzes. Alles x, welchem der Begriff des Körpers (a+b) zukommt, dem kommt auch die Anziehung (c) zu, ist ein Exempel eines synthetischen Satzes.”

2. $\forall x((Ax \wedge Bx) \rightarrow Cx)$

And when he adds:

Synthetic propositions add to cognition *materialiter*, analytic propositions merely *formaliter*,¹²

we might well be tempted to conclude that Kant-analyticity is a matter of (logical) form. Some hardy souls might even want to go so far as to say that Kant-analytic propositions are *formally* or (what amounts to the same!) *logically* true, and that their apriority and necessity stem from truth in virtue of form.

Further support for this view may be found in Kant's identification of the Law of [non-]contradiction as on the one hand "the highest principle of analytic judgments"¹³ and on the other as one of the *formal* criteria of truth.¹⁴

Yet in §2 of the *Prolegomena*, we seem to be told that the distinction between analytic and synthetic judgments concerns their content:

[...] whatever be their origin or logical form, there is a distinction in judgments, as to their content, according to which they are either merely explicative, adding nothing to the content of the cognition [by which he seems to mean the subject-idea], or ampliative, increasing the given cognition: the former may be called analytic, the latter synthetic.¹⁵

This remark seems more in line with the view that Kant's well-known table is the last word on the logical form of judgments, for the difference between (1) and (2) is neither one of quantity (both are universal), nor of quality (both are affirmative), nor of relation (both are categorical), nor finally of modality (both are assertoric).¹⁶

We find further support for this view when we turn again to the Jäsche Logic. With respect to categorical judgments—and these are the only type for which Kant

¹²*Ibid.* [Logik, AA 09.111.12-13]: "Die synthetischen Sätze vermehren das Erkenntniß *materialiter* die analytischen bloß *formaliter* [...]"

¹³KrV, A151/B191; *Prolegomena*, §2 [Prol, AA 04.267.4-5].

¹⁴Logik, AA 09.52.34-53.2.

¹⁵Prol, AA 04.266.16-23: "[...] Urtheile mögen nun einen Ursprung haben, welchen sie wollen, oder auch ihrer logischen Form nach beschaffen sein, wie sie wollen, so giebt es doch einen Unterschied derselben dem Inhalte nach, vermöge dessen sie entweder bloß *erläuternd* sind und zum Inhalte der Erkenntniß nichts hinzuthun, oder *erweiternd* und die gegebene Erkenntniß vergrößern; die erstern werden *analytische*, die zweiten *synthetische* Urtheile genannt werden können." Note also that the previous definition was framed only for categorical judgments, while this one is expressly said to apply to judgments of all forms.

¹⁶Perhaps someone will wish to object here that analytic judgments, because they are *a priori* according to Kant, might also be deemed apodeictic. Against this, recall that the same might also be said for *a priori* synthetic judgments.

defines “analytic”—we are told in §24 that the subject and predicate are the matter and the copula is the form.¹⁷ Since the only difference between (1) and (2) is that they have different predicates, it seems to follow that the difference concerns their matter and not their form. Unfortunately, things are not quite so simple: for elsewhere Kant allows that what is in one respect matter may in another be form. I have in mind here what he says about disjunctive and hypothetical judgments. Categorical judgments, he claims, are the matter of these.¹⁸ But clearly, given that there is a form/matter distinction for categorical judgments, the copula must be considered both material and formal—formal insofar as it is part of a categorical judgment, material insofar as this categorical judgment occurs within a disjunctive or hypothetical judgment.

Thus we need to know whether Kant distinguishes form and matter with respect to the subject- and predicate-concepts of categorical judgments. If so, we might still be able to maintain that the difference between (1) and (2) is one of form rather than of matter. Unfortunately, when we turn to the appropriate place in the Jäsche Logic for enlightenment on this score, we find only this:

In every concept, form and matter are to be distinguished. The matter of a concept is the object, its form is generality.¹⁹

II.

Bolzano saw the vagueness and evasiveness of Kant and his followers at such points not just as a matter of sloppiness but rather as symptomatic of a great emptiness at the heart of their explanation of the special status of logic. Attempting to introduce some order into his discussion of these matters, he begins in §81 of the *Theory of Science* by distinguishing several senses of the terms “form” and “matter”. In many cases, he remarks, “form” refers to the way the parts of a complex whole are combined. Thus, for example, a pile of building materials would differ from a house constructed with them in respect of form.

In this sense, the ideas:

- A learned son of an ignorant father
- An ignorant son of a learned father

as well as the propositions:

¹⁷Logik, AA 09.105.3-6.

¹⁸Jäsche Logic, §25 [Logik, AA 09.105.16-17], §28 [AA 09.106.22-24].

¹⁹§2; [Logik, AA 09.91.23-25]: “An jedem Begriffe ist Materie und Form zu unterscheiden. Die Materie der Begriffe ist der Gegenstand, die Form derselben die Allgemeinheit.”

- Some politicians are not men.
- Some men are not politicians.

although containing the same constituents, would differ in the arrangement of these constituents, and thus in terms of form.

If logic is sometimes called a purely formal science, it had better not be in this sense, Bolzano claims, for, to consider but one example, the concepts “not” and “some”, which occur in the two propositions above, would then fall outside the scope of logic, which is surely not what the Kantians intended.²⁰

In a second sense, Bolzano claims, “form” is taken to be equivalent to “kind” or “species”.²¹ In a way, logic can be said to be a formal science in this sense, since, generally speaking, it formulates theorems not about individual propositions, ideas, inferences, etc., but rather about entire classes of them.²² Instead, for instance, of stating that the argument:

Socrates is human.
All humans are mortal.
So Socrates is mortal.

is valid, logicians would say that any argument of the form:

X is A
All A are B
X is B

is valid (where X is to be replaced by a singular term, A, and B by general terms).

Logicians often deal with forms by means of linguistic expressions. Thus beginning with a sentence such as

Some people are virtuous.

We can replace “people” and “virtuous” with letters to produce the expression:

Some A are B.

—a linguistic form which, along with a stipulation concerning which substitutions for A and B are permissible, determines a class of sentences including:

- Some dogs are vicious.

²⁰WL, §116 [I.540].

²¹In support of this somewhat unorthodox view, Bolzano notes (WL, §81, note 1 [I.391]) that Cicero took the terms *forma* and *species* to be interchangeable.

²²WL, §12 [I.48].

- Some Greeks are philosophical.
- etc.

To the extent that the structure of these linguistic objects reflects that of the propositions they express, one can also say that the form “Some A are B” determines not only a class of sentences, but also a class of propositions. This observation motivates a third sense of “form”, according to which “form” refers to a linguistic expression that determines a kind of propositions (or ideas, arguments, etc.).²³

Note that since we can always choose to consider a different selection of constituents to be variable, it makes no sense in general to speak of a unique form of, say, a proposition or an argument. Consider, for example, the proposition:

Aristotle is wise and Achilles is vain.

By considering various parts of this proposition variable, we could arrive at a number of different forms, among them:

- X is wise and Achilles is vain.
- X is P and Achilles is vain.
- X is P and Y is vain.
- X is P and Y is Q.
- Aristotle is wise * Achilles is vain.
- X is P * Y is Q.
- A and B.
- A * B.

(where X, Y mark places for singular ideas, P, Q for general ideas, A, B for propositions and * for a binary propositional connector).

In more complex propositions, one might distinguish hundreds or even thousands of forms. Since there seems to be no upper bound on the complexity of propositions, it is more than likely that the number of forms is infinite.²⁴ Not only are these forms not in us in the way the Kantians had supposed, there will also be

²³WL §81 note 2 [I.393]; cf §12 [I.48].

²⁴Cf. WL, §186 [II.252].

a great many that can never be in us on account of their complexity and the limitations of our cognitive abilities. We do not impose these forms upon the objects of our thought—I take this to be one of the central thrusts of Bolzano’s claim that propositions and ideas in themselves are prior to thought—it would be more accurate to say that they impose themselves on us. If so, the Kantian arguments for the apriority, necessity and certainty of logic can no longer stand.

We are all used to hearing Kant compared to Copernicus. Yet on the topic of logical form, it is Bolzano who plays Copernicus to Kant’s Ptolemy. For Kant, man is at the centre of things with respect to logical form, and the universe he surveys is relatively small and completely open to his view. Bolzano’s conception, on the other hand, puts logical form on a footing similar to that of the physical world: independent of us, far too large for us to understand completely, it is nevertheless something we can, under certain circumstances, come to know at least in part.

III.

With his more precise notion of propositional form in hand, Bolzano was able to formulate a workable definition of truth in virtue of form.²⁵ Looking at a typical example of an analytic proposition in Kant’s sense, for example:

(*) A man who is married is married.

Bolzano recognised that while one could say that its analyticity depended on its content (i.e., the fact that “married” occurs in both subject and predicate), it was far more fruitful to look upon analyticity as a matter of form—here, the propositional form:

An A, which is B, is B.

For him, the interesting thing here was the invariance of truth-value under variation of some parts of a proposition: not only is (*) true, it also remains true whenever we substitute appropriate ideas for “man” and “married”.

Before proceeding, I must touch upon a small point of detail, namely, the question of which substitutions are appropriate. Clearly, no matter what propositional form we are talking about, not all substitutions will result in true propositions. In the proposition

The man Caius is mortal.

²⁵WL, §147.

for instance, substitution of the idea “Frege” for “Caius” will result in another true proposition, but the substitution of “Helsinki”, “and”, or “virtue” will not.

In Bolzano’s opinion, this problem arises even in the case of propositions such as

A married man is married.

since the substitution of “round” for each of the two occurrences of “married” and “square” for “man” results in the proposition:

A round square is round.

which he, in line with logical tradition and common usage, counts as false because its subject-idea represents no object.²⁶

Bolzano introduces the term “objectual” [*gegenständlich*] for ideas that have objects and the term “objectless” for those that do not.²⁷ A *proposition* is then said to be objectual if its subject-idea is. He then proposes the following modification to the above remarks.²⁸ What is special about propositions which are analytic in Kant’s sense, i. e., those of the form

An A, which is B, is B.

is that every *objectual* instance of the form is true.

With this proviso in mind, we can say that one of the most interesting features of Kant-analytic propositions is that their truth-value remains constant under an entire class of transformations. Once we see things in this light, however, there is no reason to single out Kant’s case for special attention. For invariance of truth-value under a class of transformations is clearly a far more general phenomenon. Propositions such as

If $e < 3$ and $3 < \pi$, then $e < \pi$

²⁶Examples such as “A round square is round”, incidentally, show in Bolzano’s opinion that propositions that are analytic in Kant’s sense may be false. Thus to know that a Kant-analytic proposition is true, we must first know that its subject-concept has an object. But this latter proposition is an existential claim and thus, by Kant’s own lights, synthetic (KrV, A 598/B626). Thus knowledge of Kant-analytic truths presupposes knowledge of synthetic truths. See WL, §305.1 [III.178]. Based on what he says in the Jäsche Logic, Kant himself would have had to agree with these remarks, since he maintained there that universal propositions do have existential import. See, for example, §46 [Logik, AA 09.116.7-8], where K. quotes with approval the canon: “The inference from the universal to the particular is valid (*ab universali ad particulare valet consequentia*).” [Vom allgemeinen gilt der Schluß auf das Besondere.]

²⁷WL, §50 [I.222].

²⁸WL, §146 [II.77].

for example, also have this property, since uniform variation of all occurrences of “e”, “3” and “π” does not change the truth value. So too the proposition

$$2 < 3 \text{ and } 3 < 2$$

is not only false, but remains so no matter what ideas we may substitute for “2” and “3”.

Thinking he has put his finger on the most important insight in Kant’s reflections on analyticity, Bolzano gives the following extremely general definition of truth in virtue of form, or *universal validity*, as well as falsity in virtue of form or *universal invalidity*:

Let the proposition *A* be such that all the propositions which can be generated from it are true, if the ideas *i, j, . . .* alone are considered variable, and if only objectual propositions may be formed. Then [. . .] we can call the proposition *universally* or *fully valid*. If, on the other hand, all propositions developed from *A* are false, [. . .] [we can] say that it is a *universally* or *absolutely invalid* proposition. Universally valid propositions could also be said to be *true after their kind* or *form*, universally invalid propositions *false* after their kind or form, where by *kind* is meant the sum of all propositions, which differ from *A* only in the ideas *i, j, . . .*²⁹

He then proposes that we apply the term “analytic” to any proposition that is true (or false) in virtue of at least one of its forms.³⁰

Note too that Bolzano-analyticity is quite closely connected with *deducibility* (or consequence) in Bolzano’s understanding. According to the definition given in §155 of the *Theory of Science*, a proposition *X* is deducible from certain others *A, B, C, . . .* relative to the variable parts *i, j, k, . . .* iff every appropriate substitution for *i, j, k, . . .* that makes all of *A, B, C, . . .* true also makes *X* true. It follows that if a proposition *X* is deducible from others *A, B, C, . . ., M* with respect to variands *i, j, k, . . .* then the proposition

If (*A* and *B* and *C* . . . and *M*) then *X*

is universally valid w.r.t. the same variands, and conversely.

Here are some of the examples of analytic propositions (or universally satisfiable/unsatisfiable propositional forms) that Bolzano cites (the variands are indicated either by underlining or by letters):

²⁹WL, §147 [II.82].

³⁰WL, §148.

1. The man Caius is mortal. (WL, §147)
2. The man Caius is omniscient. (WL, §147)
3. A is A. (WL, §148)
4. An A which is a B is an A. (WL, §148)
5. An A which is a B is a B. (WL, §148)
6. Every object is either B or non-B. (WL, §148)
7. If all men are mortal and Caius is a man, then Caius is mortal. (WL, §315)
8. If A is larger than B, then B is smaller than A. (WL, §148)
9. If $P = Mm$, then $M = \frac{P}{m}$. (WL, §148)
10. The soul of Socrates is a simple substance. (WL, §447)
11. If $\frac{a^2}{2} = b$, then $a = \pm\sqrt{2b}$ (WL, §447)

Bolzano singles out cases such as numbers 3-7 for special attention. Only logical knowledge is required to see that they are analytic, he claims, because the only invariable parts of the relevant forms are logical concepts. He proposes that we call such propositions *logically analytic*.³¹

IV.

In recasting the Kantian definition in this way, Bolzano also drained the concept of analyticity of any special epistemological or metaphysical significance, for analyticity in his extremely general sense entails neither apriority nor necessity. The following proposition, for example, would count as analytic, where the underlining indicates the variable parts:

12. If this man is Napoleon, then this man was victorious at Marengo.

This example will also help us to appreciate the distance between Bolzano's conception and Kant's. For Bolzano, (12) is certainly analytic, but not because (as Kant maintained) its subject contains its predicate (for it is not even a categorical proposition), nor because (as Leibniz maintained) if a concept is really to be a

³¹WL, §148 [II.84].

concept of Napoleon, it must somehow (virtually) contain the predicate “was victorious at Marengo”. Rather, it is simply because “If $x = \text{Napoleon}$, then x was victorious at Marengo” is true for every x .

Because the Bolzano-analyticity of (12) is contingent and cannot be recognized *a priori*, one might be tempted to speak here of *material* analyticity, on the model of so-called material consequence. But this would be to smuggle in the very Kantian assumptions that Bolzano explicitly rejects. For Bolzano, all analyticity, all consequence, is by its nature formal. Saying this, however, by no means commits us to saying that we must always be in a position to know whether a given proposition is analytic, or a relation of consequence holds, still less that we must always be able to know this *a priori*.

It seems to me that this must even be so in the case of logical analyticity. In order to explain why, however, I will first have to say a few words about Bolzano’s understanding of apriority.

V.

Bolzano, reasonably enough, took the term “*a priori*” to apply to knowledge. To say that a (true) *proposition* is *a priori* can then be taken to mean either that it *is* or else that it *can be known a priori*. Now Kant, along with many others, had stated that all the truths of logic and mathematics are *a priori*. This could not mean that all of them were already known, so Bolzano interpreted the claim to mean that all of them *could* be known *a priori*. Let us notice first of all that neither Kant nor anyone else had any good reason to think that this was true. But even if they did have a reason, it could not be that these propositions were already known *a priori*. There must have been some other feature of mathematical propositions that they were able to recognise which allowed them to class them together and conjecture that all the true ones can be known *a priori*.

Bolzano thought he had found a serviceable criterion in the notion of a *purely conceptual proposition*.³² For the present purpose it will suffice to say that in order to grasp a purely conceptual proposition we do not need to represent any contingently existing particulars. The remaining propositions (the so-called *intuitional* or *empirical* propositions) cannot be grasped without representing contingently existing particulars, and hence can neither be grasped nor *a fortiori* known without

³²WL, §133; see also WL, §§72-77 and Bolzano, B. *On the Mathematical Method and Correspondence with Exner*, ed. and tr. P. Rusnock and R. George. Amsterdam, 2004, 50-53. Bolzano’s account of purely conceptual propositions is based upon his notion of an *intuition*. For an illuminating comparison of Kant’s and Bolzano’s notions of intuitions, see George, Rolf, “Intuitions: the theories of Kant and Bolzano.” In: Siebel, M. and Textor, M., eds., *Semantik und Ontologie*. Frankfurt/Lancaster, 2004, 319-353.

experience. Thus if any propositions are to be known *a priori*, they will have to be purely conceptual.

Purely conceptual propositions can be expressed by sentences that contain no demonstratives, indexicals, proper names or natural kind terms.³³ Since logical and mathematical propositions all have this character, it seems reasonable to suppose that they qualify as purely conceptual. Does this mean that they are all knowable *a priori*? Not necessarily, for only the converse has been established: if a proposition can be known *a priori*, it is purely conceptual.

But what does it mean for us to know a proposition *a priori*? Bolzano offers the following definition:

If the propositions from which a judgment M is deduced, as well as those from which the former follow, down to the immediate judgments, are all purely conceptual propositions, then a judgment M can be called a judgment from pure concepts, or pure, or *a priori*. In all other cases it could be said to be drawn from experience or *a posteriori*.³⁴

In order to show that every purely conceptual proposition was knowable *a priori*, then, we would have to show that all were deducible in this way. But how could you prove *that*?³⁵

I do not mean to claim here that Bolzano somehow anticipated Gödel's incompleteness theorem. That is not the point at all. It was not that he had proved the incompleteness of our access to the collection of purely conceptual truths; rather, he knew quite well that no one had proved its completeness.

Thus it would be a mistake simply to identify purely conceptual propositions with those that can be known *a priori*. For it might well be the case that we deem a purely conceptual proposition to be true on empirical grounds, and are incapable of justifying it in any other way given the current state of our knowledge. Bolzano cites as an example Newton's law of gravitation,³⁶ but he might equally well have cited pretty much any proposition of mathematics, since it would have been highly implausible to claim that any of them had been deduced from purely conceptual first principles by means of a gapless sequence of strict inferences.³⁷

³³Cf. WL, §75.

³⁴WL, §306 [3.202]. Cf. Frege's formulation *Foundations of Arithmetic*, §3.

³⁵Frege seems simply to have assumed that all the truths of mathematics could be furnished with *a priori* proofs; see his division of *truths* into those *whose proof* requires appeal to facts and those whose proof does not (*Foundations of Arithmetic*, §3; emphasis added).

³⁶*On the Mathematical Method and Correspondence with Exner*, 53.

³⁷Dedekind, for example, could claim with justice in 1887 that "the theorem that $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ has nowhere yet been strictly demonstrated" ("The nature and meaning of numbers", in *Essays on the Theory of Numbers* tr. W. Beman [New York: Dover, 1963], 40).

Indeed, Bolzano's entire mathematical career was devoted to seeking the objective grounds for some of the most basic propositions of mathematics—a task which, if successful, would first justify the claim that at least one person knew these truths *a priori*.

Let us now return to the question of logical analyticity. Recall that a proposition is said to be logically analytic iff it is universally satisfiable relative to a form where all the invariable parts are logical ideas. Assuming that all logical ideas are pure concepts, it seems reasonable to suppose that such forms can be represented in purely conceptual terms, so that a proposition stating that every (objectual) instance of such a form has the same truth value would be a purely conceptual proposition. In some cases, at least, this interesting truth might be known *a priori*. But there is no guarantee that this must always be so.

VI.

Finally, a word on certainty. Kant had declared in the *Jäsche Logic*:

[I]n no science that contains *a priori* cognitions—thus neither in mathematics, nor in metaphysics or morals—is an *opinion* to be found. For it is in and of itself absurd to opine *a priori*. Nor could anything be more ridiculous than, for example, to merely have an opinion in mathematics. Here, as in metaphysics and morals, one either knows or does not know!³⁸

Applied to philosophy, this was just funny. Here, for instance, is Bolzano's comment on the spectacle provided by the German philosophers of his day:

Almost every one of them affects a tone of complete certitude, saying that he and *he alone* has discovered the one true system; and the most ridiculous part is that one hears this kind of talk even from philosophers who have completely changed their own systems several times.³⁹

It took a little more effort to see that Kant's remarks didn't apply any better to the so-called formal sciences of logic and mathematics.

According to the official publicity of the time, mathematics was a paradigm of certain knowledge, consisting of a collection of self-evident first truths and

³⁸Logik, AA 09.67.5-12: "Wo findet nun aber das bloße Meinen eigentlich statt? In keinen Wissenschaften, welche Erkenntnisse *a priori* enthalten; also weder in der Mathematik, noch in der Metaphysik, noch in der Moral [. . .] Denn es ist an sich ungereimt *a priori* zu meinen. Auch könnte in der That nichts lächerlicher sein, als z. B. in der Mathematik nur zu meinen. Hier, so wie in der Metaphysik und moral, gilt es: *entweder zu wissen oder nicht zu wissen*." Cf. KrV, A xv.

³⁹*Lehrbuch der Religionswissenschaft* (Sulzbach, 1834), I, §63.

the theorems derived from these by means of self-evidently truth-preserving inferences. Most philosophers were in no position to know any better, since ignorance of mathematics, despite Plato's decree, was by no means a barrier to employment. Bolzano, as we know, was different. As one of the first mathematicians who looked into the foundations of analysis in any detail, he knew quite well that the official publicity was pure invention. More clearly than most, he knew that first truths were hard to come by, that what is obvious may also be false, and that theorems might be more evident than the principles from which they must be derived.

Bolzano agreed with Kant that the doctrines of mathematics and logic were more reliable than most. But, he added

[...] I see no reason why we should forsake the explanation [of this] that had been given long before Kant. It has always been maintained that these sciences enjoy such a high degree of certainty only because they have the advantage that their most important doctrines can be easily and variously tested by experience, and have been so tested, and that those doctrines which cannot be immediately tested are deducible by arguments which have been tested many times and have always been found valid, and finally, that the results which are obtained in these sciences do not infringe upon the human passions; hence most of these investigations were begun and finished without bias, and with suitable leisure and peace. The only reason why we are so certain that the rules *Barbara*, *Celarent*, etc. are valid is because they have been confirmed in thousands of arguments in which we have applied them. [...] If we have not tested the truth of a proposition either by experiment, or by repeated checking of its derivation, we do not give it unqualified assent, if we are at all sensible, no matter what Critical Philosophy may say [...].⁴⁰

Bolzano may have developed his distaste for talk of infallibility in his day job as professor of the Catholic religion at the Charles University. Most likely another habit of his was formed there as well: that of striving to find common ground even with the people he disagreed with most profoundly. We have seen that Bolzano bent over backwards to interpret what the Kantians said in such a way that he could agree at least verbally with some of their claims. The result of these efforts is an appearance of substantial agreement on fundamental matters. Bolzano adopts Kant's terms "analytic" and "synthetic" as his own, and certainly would be willing to say that there is such a thing as truth in virtue of form, which might in some cases be known *a priori*. But the appearance of consensus is misleading.

⁴⁰WL, §315 [III.244-245].

Because of the many absolutes laid down in Kant's discussion of logical form and analyticity, to disagree even partially with him often amounts to a complete rejection of his claims—and we have seen that this was the case right across the line in Bolzano's response. Faced with his invariable reasonableness and measured tone, we all too easily miss the vast gulf separating Bolzano from the Kantians, and may not even notice that his disagreements are neither piecemeal nor *ad hoc*, but rather are systematically rooted in his own, equally radical philosophical vision.