Kant and Bolzano on Analyticity

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> Dieser Weltweise [*sc.*, Kant] hatte (so glaube ich) bei vielem *Tiefsinn*, d.i. bey der Gabe auch tief Verborgenes dunkel zu fühlen, doch nicht die Gabe des *deutlichen Denkens* [....] (Bolzano to Florian Werner, **BBGA** Series III, Vol. 5/1, p. 175.)

> [T]he word 'analytic' is a philosopher's term of art. Memories of doctrines associated with this term (be they Kantian, Fregean, Carnapian, or whatever) should not be mistaken for pre-theoretical 'intuitions' concerning analyticity. There simply are no such intuitions one could appeal to. (W. Künne, "Analyticity and logical truth: from Bolzano to Quine," p. 219)

Introduction

The history of speculation around a notion or notions called analyticity is often viewed from the perspective of the dispute between Carnap and Quine. Previous characterizations, due to Kant, Frege and others, are then seen as being of a piece with, and leading up to, Carnap's various definitions of analyticity, and hence as directly or indirectly subject to Quine's objections. One thus gets the impression of an unbroken tradition of philosophical reflection on a single notion (or closely related cluster of notions) of analyticity, however imperfectly it may have been grasped by some of those involved. By implication, at least, this seems to be the view of many contemporary philosophers, who speak of *the* analytic/synthetic distinction in the singular, as the common topic of Kant, Frege, Carnap and others.[1]

What is this traditional notion of analyticity? According to Quine, it may be loosely characterized as truth by virtue of meanings and independently of fact.[2] How, exactly, this is to be made precise remains an open question, but at least this much seems to be generally granted: analytic propositions are both necessary and knowable *a priori*.[3] Moreover, analyticity is apt to be invoked in attempts to *explain* at least some and perhaps all cases of *a priori* knowledge and necessary truth.

Those who accept some version of this narrative will find that Bolzano constitutes an anomaly, as his notion of analyticity is decidedly non-linguistic, and completely independent of both apriority and necessity. If the traditional notion of analyticity were as just described, he would have to be regarded as singularly eccentric, willful, or obtuse in failing to recognise its essential features. These are indeed conclusions that some commentators have reached. I shall argue, on

[1] See, e.g., Rey 2008.
[2] See, e.g., Quine 1980: 20.
[3] See., e.g., Quine 1960: 67.

the contrary, that the problem lies not with Bolzano but rather with the historical account sketched above. I begin, as one might expect, with Kant.

Kant and his precursors

Kant's remarks on the analytic/synthetic distinction present a number of challenges to interpreters. In the *Prolegomena*, to begin with, he speaks of synthetic judgments [*Urtheile*] and propositions [*Sätze*], but also of synthetic cognitions [*Erkenntnisse*], creating some doubt concerning just what sort of entities are supposed to be classified as analytic or synthetic. In the Jäsche *Logic*, identical propositions or tautologies are said to be analytic,[4] while in a late essay Kant denies this.[5] In the *Critique of Pure Reason*, we seem to be told that the distinction applies only to categorical judgments, while in the *Prolegomena*, the distinction is claimed to apply to judgments of any logical form.[6] Finally, we find several different definitions of 'analytic judgment' in his writings. In the *Critique of Pure Reason*, we read that:

> In all judgments in which we think the relation of a subject to the predicate (I here consider affirmative judgments only, because the application to negative judgments is easy afterwards), this relation is possible in two ways. Either the predicate B belongs to the subject A as something that is (covertly) contained in the concept A; or B, though connected with concept A, lies quite outside it. In the first case I call the judgment *analytic*; in the second *synthetic*.[7]

In the *Prolegomena*, we have:

> [W]hatever be their origin or logical form, there is a distinction in judgments, as to their content, according to which they are either merely *explicative*, adding nothing to the content of the cognition, or *ampliative*, increasing the given cognition: the former may be called *analytic*, the latter *synthetic*, judgments.[8]

While in the Jäsche Logic, we find still another definition:

> Propositions whose certainty rests on *identity* of concepts (of the predicate with the notion of the subject) are called *analytic* propositions. Propositions whose truth is not grounded on identity of concepts must be called *synthetic*.[9]

[4] Jäsche Logic, §37 [**Ak.** 9: 111].
[5] **Ak.** 20: 322.
[6] A6/B10; *Prolegomena*, §2 [**Ak.** 4: 266].
[7] A6/B10. Except as otherwise noted, I use Werner Pluhar's translation (Kant 1996).
[8] **Ak.** 4: 266; tr. from Kant, 1977. Cf. Kant 1790 (**Ak.** 8: 228).
[9] §36 [**Ak.** 9: 111]; tr. from Kant 1992a.

Some commentators have claimed to find yet another definition in Kant's presentation of the "Supreme Principle of All Analytic Judgments".[10] In this part of the *Critique*, Kant informs us that it is none other than the principle of contradiction, namely, "No thing can have a predicate that contradicts it [....]" The connection with analytic judgments is relatively straightforward:

> *[I]f a judgment is analytic,* whether it be negative or affirmative, then its truth must always be cognizable sufficiently by reference to the principle of contradiction. For denying the reverse of what already lies, and is thought, as concept in the cognition of the object will always have to be correct; but the concept itself will necessarily have to be affirmed of the object, because the opposite of the concept would contradict the object.[11]

That is, if we begin with an analytic judgment, say "Dishonest politicians are politicians", and transform its predicate into its opposite, we obtain the self-contradictory judgment: "Dishonest politicians are not politicians". By the principle of contradiction the latter is false, hence the original judgment is true. Kant's remarks, it seems to me, should not be stretched. This is not a new, wider definition of analyticity, but rather a straightforward claim concerning the basis of truth of analytic judgments as Kant has defined them.

No doubt there are problems enough here to keep commentators guessing about Kant's true intentions in framing his distinction. Nonetheless, it seems to me that he is sufficiently clear to permit us to say the following with some confidence.

(1) It is not *sentences* or any other linguistic entities that are to be classified as analytic or synthetic, but rather *judgments*. Kant's talk of synthetic '*Sätze*' might lead one to think otherwise. But although many of his contemporaries used '*Satz*' to designate sentences, i.e., verbal expressions of judgments, Kant reserved the term for assertoric judgments.[12] Since linguistic entities are not in play, interpreting Kant-analyticity as truth in virtue of meaning involves a certain amount of contortion.

What of cognitions? According to the *Critique of Pure Reason*, cognitions [*Erkenntnisse*] are a kind of presentation [*Vorstellung*], namely, objective perceptions, the species of which are concept and intuition.[13] On the face of things, this would rule out judgments, since, according to Kant, concepts and intuitions are the presentations that serve as subjects and predicates in judgments. Yet elsewhere in the same work, he tells us that judgments *are* a kind of cognition:

[10] Kant 1781/1787: A150/B189 *et seq.*
[11] A151/B190-191.
[12] See, e.g, **Ak.** 8: 193, note; Jäsche Logic, §30, note 3 [**Ak.** 9: 109]; Vienna Logic, **Ak.** 24: 934.
[13] A320/B376-77

> Judgment [...] is the mediate cognition of an object, viz., the presentation of a presentation of it.[14]

which, as odd as it may sound to modern readers, puts judgments squarely in the same category as their terms. Thus when he asked about the possibility of synthetic cognitions, he may simply have been thinking of judgments. On the other hand, he may also have been thinking about synthetic cognitions in a different sense of the term (i.e., complex presentations of objects assembled by *a priori* synthesis). But this is clearly a different sense of 'synthetic' than that exposed in his various definitions (for in a mere presentation, there is no predicate, no subject, no possibility of containment of one in the other). Either way, I do not think that the question "Wie sind synthetische *Erkenntnisse a priori* möglich?" forces us to extend the domain of application of the terms 'analytic' and 'synthetic' in the sense we are concerned with here beyond judgments.

(2) Despite the gesture towards judgments in general in the *Prolegomena*, the analytic/synthetic distinction is only defined for *categorical* judgments.[15]

In saying this, I am fully aware of the long history of regarding the distinction between merely *explicative* and *ampliative* judgments as a basis for a broader definition. In my view, however, this interpretation is not justified. Note that he says that in an analytic judgment, nothing is added to the content of *the* cognition, while in synthetic judgments, something is added to *the given* cognition.[16] *Which* cognition, one might ask? Elsewhere, it is made clear that it is the *subject* of the judgment that is meant. Compare what he has to say in the *Critique of Pure Reason*:

> Analytic judgments could also be called *explicative*. For they do not add anything to the concept of the subject through the predicate [....] Synthetic judgments, on the other hand, could also be called *ampliative*. For they do add to the concept of the subject a predicate that had not been thought in that concept at all [....][17]

Kant's point has been obscured by a semantic shift in German since the eighteenth century. As mentioned above, Kant generally uses '*Erkenntniß*' to refer to a kind of presentation, namely, objective perceptions; it is the genus of which concept and intuition are the species.[18] *Erkenntnisse* in this sense are the represen-

[14] A68/B93: "Das Urtheil ist ... die Mittelbare Erkenntniß eines Gegenstandes, mithin die Vorstellung einer Vorstellung desselben."

[15] More specifically, Kant appears to tailor most of his remarks to cover the special case of universal or singular affirmative categorical judgments.

[16] **Ak.** 4: 266.

[17] A7/B11. In the same passage, K. also makes clear the connection between the "subject contains predicate" conception of analyticity and truth (or certainty) "based upon the identity of concepts"

[18] A320/B376-377. Cf. Adelung 1808, Arts. "Erkennen", "Erkenntniß", and the illuminating discussion in George 1982.

tations that serve as the terms (subjects and predicates) of categorical judgments. Since Kant's time, this usage has all but disappeared from the German language; '*Erkenntnis*', in many of its uses, is now closer in meaning to 'knowledge', and has been translated as such, notably by Kemp Smith (today, Kant's translators prefer 'cognition'). This rendering often produces quite satisfactory results, but not, as Rolf George points out, for the reason one might expect:

> Translation of *Erkenntnis* as "knowledge" is appropriate much of the time, not because Kant used the word in the contemporary sense, but because, quite generally, knowledge was then thought to be a felicitous kind of representation, a sort of successful reference. We might call it the Adamic Language Theory of knowledge: If one represented an object in one's mind by a kind of token that was really fitting, in the way in which the names that Adam gave to things were the real names of things, then one was thought to be as close to knowing the thing as was humanly possible. This makes understandable the close connection in eighteenth-century philosophy between good reference and knowledge: To know is to have a good picture, the right concept, the correct name, of a thing.[19]

In the present case, however, rendering '*Erkenntnis*' by 'knowledge' is decidedy misleading. It would have Kant say that synthetic judgments are those which increase our *knowledge*, while analytic judgments are those which do not.[20] In this vein, Coffa, for example, writes:

> The heart of the problem is Kant's seemingly harmless assumption that the analytic-synthetic distinction is a correct explication of another one, between clarificatory judgments (*Erläuterungsurteile*) and ampliative judgments (*Erweiterungsurteile*). In all likelihood, Kant never realized that he was dealing with two different distinctions. Thus, some of his "definitions" of analytic and synthetic judgments tell us that the latter "extend my knowledge beyond what is contained in the [subject] concept" [...] But it is essential to realize that we are dealing here with a second partition of the class of all (true, subject-predicate) judgments into those that we may ground, or identify as true, merely on the basis of the fact that we are clear about the concepts involved in the judgment, and those (other judgments) that call for an appeal to extra-conceptual sources of knowledge. Roughly speaking, whereas Kant's first, *nominal*

[19] George 1982: 35.

[20] This was common practice in earlier English translations of Kant's works. See, e.g., Kant 1965: A8: "through analytic judgments our *knowledge* is not in any way extended"; Kant 1950: §2a: "there is a distinction in judgments [...] according to which they are either merely *explicative*, adding nothing to the content of *knowledge* [...]"; Allison 1973: 151: "This predicate [*sc.*, in a synthetic judgment] [...] extends my *knowledge* beyond what is contained in that concept [*sc.*, the subject concept]."

> definition characterizes "analytic" as true in virtue of definitions (analysis) and logic, the second one defines it as true in virtue of meaning.[21]

Truth in virtue of logic and definitions and truth in virtue of meaning may well be different things, as Coffa suggests. Yet I cannot agree with his contention that both notions are adverted to in Kant's remarks. Rather, I believe, the characterizations "predicate contained in subject" and "merely explicative" (adding nothing to *the* cognition, i.e., to the subject concept) were in fact meant to amount to the same thing.

The problem is not just one of translation; Frege also seems to have missed Kant's point. In §88 of the *Foundations of Arithmetic*, for example, speaking of consequences drawn in mathematics with the help of definitions such as that of the continuity of a function, he writes:

> Diese Folgerungen *erweitern unsere Kenntnisse*, und man sollte sie daher Kant zufolge für synthetisch halten; dennoch können sie rein logisch bewiesen werden, und sind also analytisch.[22]

which seems to assume that Kant thought that analytic judgments do not increase our knowledge. Yet I am not at all convinced that Kant ever thought that. Indeed, in the Jäsche Logic, we find a remark that suggests otherwise: both analytic and synthetic judgments, we are told there, "*vermehren das Erkenntniß*", analytic judgments merely *formally*, synthetic ones *materially.*[23]

How does an analytic judgment advance our knowledge? By making us aware of what we already think, though confusedly, i.e., by rendering a concept more distinct. A transcription of Kant's logic lectures finds him illustrating a similar point with respect to an intuitive representation:

> If, in a composite representation, I distinguish from one another the parts of which it consists, that is a distinct representation. But if I do not distinguish its partial representations from one another, then it is a *confused representation.* E.g., if I see a cheese mite, my representation of it is in the beginning confused. But if I take a *microscopium* in hand and become aware in it of a jaw full of teeth, 2 rows of feet, 2 black eyes, then I represented all of that previously. But I was not able to distinguish these partial representations of the limbs from the whole body.[24]

Note his remark: *I represented all of that previously.* For if I saw the whole mite, he reasoned, I must have seen its parts, hence the teeth, the feet, etc. The same point

[21] Coffa 1992: 16. Underlining added.
[22] Frege 1884: 101. Emphasis added.
[23] §36, note 1 [**Ak.** 9: 111].
[24] From the Logik Blomberg, **Ak.** 24/1: 41 (Kant 1992a: 28).

is made in the *Anthropology*, where he remarks that if I see a man's face, but am not aware of seeing his nose and eyes, etc. (perhaps viewing him from a distance) I must nonetheless have formed representations of that nose and those eyes:

> For if I wanted to maintain that I do not at all have the representation of him in my intuition because I am not conscious of perceiving these parts of his head [...] then I would also not be able to say that I see a human being, since the representation of the whole (of the head or the human being) is composed of these partial ideas.[25]

It follows that there are a great many representations in our minds of which we are unaware (very few if any of them having to do with oedipal longings). In a sense, then, we know that the man has a nose (we represent it), but in another sense we do not, since we are not aware of this. In becoming aware of such details, similarly, in a sense we learn nothing new, but in another we may learn quite a lot.

Things are much the same in the case of the analysis of confused concepts. Even a simple farmer who applies the concept of justice has in his mind, Kant thinks, all the marks belonging to that concept, even though he most likely has no awareness of this mass of detail. Analysis, the peculiar occupation of the philosopher, brings it to light:

> [D]istinctness in concepts [...] involves the analysis of what I think, what I already actually conceive in thoughts, i.e., the distinctness of the understanding *per analysin*.[26]

An analytic judgment is the act whereby such distinctness is produced. A judgment that does not do this, even if it is of the form 'A is A', Kant suggests elsewhere, does not really deserve to be called analytic. Rather, it should be called empty:

> [...] such judgments contribute nothing to the distinctness of the concept, something at which all judging must aim, and are accordingly called empty, e.g., every body is a bodily (in other words, material) entity. Analytic judgments are indeed grounded in identity, and can be resolved into it, but they are not identical, for they require analysis and thereby serve to define the concept; by contrast, with identical judgments, something is defined by the very same thing, *idem per idem*, and hence nothing is defined.[27]

The point is elaborated, following a slightly different trajectory, in another transcription of Kant's logic lectures, where he observes that propositions such as 'a human has a human nature'

[25] **Ak.**, 7: 135 (Kant 2006: 24).
[26] **Ak.** 24/1: 42 (Kant 1992a: 29).
[27] **Ak.** 20: 322.

> ... promote cognition neither analytically nor synthetically, they are tautological propositions; by means of them I have neither an increase in distinctness nor an addition to the cognition. But not all identical propositions are empty, only the tautological ones. An identical proposition that can only be understood through analysis should not be looked upon as an empty proposition: for it furthers distinctness.[28]

And this increase in distinctness *is* an advance in our knowledge.

(3) Kant's talk of containment is literal. In analytic judgments, the concept of the predicate is literally a part of the concept of the subject. It seems to me that Kant's definitions, as well as his examples and further explanations, make this abundantly clear. In the *Critique of Pure Reason*, for instance, we read:

> [I]f I say: All bodies are extended—then this is an analytic judgment. For I do not need to go beyond the concept that I link with the word body in order to find that extension is connected with it. All I need to do in order to find this predicate in the concept is to dissect the concept, i.e., become conscious of the manifold that I always think in it. By contrast, if I say: All bodies are heavy—then the predicate is something quite different from what I think in the mere concept of a body as such. Hence adding such a predicate yields a synthetic judgment.[29]

The Jäsche Logic is, if anything, more explicit:

> An example of an *analytic* proposition is, To everything x, to which the concept of body $(a + b)$ belongs, belongs also *extension* (b).
> An example of a *synthetic* proposition is, To everything x, to which the concept of body $(a + b)$ belongs, belongs also *attraction* (c).[30]

In case this is not enough, we have the following:

> [I]n an analytic proposition the question is only whether I actually think the predicate in the presentation of the subject.[31]

Despite what seems to me to be remarkable clarity on Kant's part, there has been a persistent tendency to think that he must have meant something else. Frege, as is well known, implausibly claimed his quite different characterization of analyticity to be a mere clarification of Kant's.[32] Arthur Pap, in a similar vein, tells us that "it has often been pointed out that Kant's definition of an analytic judgment

[28] From the Logik Busolt, **Ak.** 24/2: 667.
[29] A7/B11.
[30] §36 [**Ak.** 9: 111].
[31] A164/B205.
[32] Frege 1884: §3. He is less generous, and more accurate, in §88 of the same work.

[. . .] is unsatisfactory [in part because] the literal meaning of the metaphor 'contained' is not clear [. . . .]"[33] He proceeds to claim that "Kant's intention, no doubt, was to say that an analytic proposition is a proposition from whose negation a contradiction, i.e. a statement of the form 'p and not-p' is deducible." Quine, similarly, deemed Kant's talk of containment to be metaphorical, and offered the following:

> Kant's intent, evident more from the use he makes of the notion than from his definition of it, can be restated thus: a statement is analytic when it is true by virtue of meanings and independently of fact.[34]

The reluctance to take Kant at his word may well be due to the apparent triviality of his distinction when it is viewed from the modern perspective. Kant had claimed in the *Prolegomena*, for example, that one of his greatest discoveries was that (almost) all mathematical judgments were synthetic:

> *Mathematical judgments* are all synthetic. This fact seems hitherto to have altogether escaped the observation of those who have analyzed human reason; it even seems directly opposed to all their conjectures [. . . .][35]

Was Kant—often said to be the greatest of the modern philosophers—really claiming as an epoch-making discovery the observation that not all mathematical truths had the same form as "A red apple is red"? This seems an odd boast, to say the least, so it is perhaps not surprising that many of Kant's modern readers have applied the Kripke transform to his remarks:

> Kant says [. . . .] If he says that, then it's especially strange, so let's suppose that's not what he's saying.[36]

But there is no call for such drastic measures. For, as it turns out, a number of celebrated eighteenth-century German philosophers did in fact hold the view that Kant thought he was the first to reject.

Moses Mendelsohn, for example, wrote the following in an essay that beat Kant's by a nose to take the Berlin Academy's prize in 1764:

> Mathematics grounds its certainty on the universal axiom that nothing can be and not be simultaneously. In this science, one proves every proposition, such as **A** is **B**, in one of two ways. Either one unfolds [*entwickelt*] the concept of **A** and shows that **A** is **B**, or one unfolds the concept of **B** and shows that non-**B** must also be non-**A**.

[33] Pap 1958: 27.
[34] Quine 1980: 21.
[35] Kant 1783, §2.c.2 (**Ak.** 4: 268).
[36] Kripke 1980: 117.

> Both sorts of proof are thus based on the principle of contradiction, and since the object of mathematics in general is *quantity*, and the object of geometry in particular is *extension*, one may say that in mathematics the concept of quantity, in geometry the concept of extension, is unfolded and laid open [*auseinandergesetzt*]. In fact, there is no other foundation for geometry than the abstracted concept of extension, and from this single source all the consequences of the science are drawn, and indeed in such as way that one distinctly recognises that every claim of geometry is necessarily connected to the fertile concept of extension through the principle of contradiction.[37]

In his *Path to Certainty*, Crusius makes a similar claim:

> Pure *mathesis* nowhere requires any other *Principium* than the mere principle of contradiction. For it considers either existential-*abstracta* or *Principiata* which depend upon *principiis existentialiter determinantibus*. These must necessarily be recognizable from the defined quantitative entities merely by means of the principle of contradiction.[38]

And of course the great Leibniz himself had said many similar things:

> The great foundation of mathematics is the *principle of contradiction or identity*, that is, that a proposition cannot be true and false at the same time and that therefore A is A and cannot be non-A. This single principle is sufficient to demonstrate every part of arithmetic and geometry, that is, all mathematical principles.[39]

Such claims did not stop at mathematics. Leibniz had announced that there were two great principles of all our reasoning: the principle of contradiction and the principle of sufficient reason.[40] Seeking to outdo their great predecessor, various German philosophers had attempted to reduce these two principles to one, by proving the principle of sufficient reason from the principle of contradiction. In the *Prolegomena*, Kant mentions Wolff and Baumgarten as misguided philosophers who sought thus to prove this "clearly synthetic" principle.[41] Later, he would take Eberhard to task for the same thing.[42] He did not mention his younger self, though he certainly might have, for in his first publication, Kant sought to improve upon the proofs of Wolff and Baumgarten.[43]

[37] Mendelsohn 1838: 294.

[38] Crusius 1747: §10 [p. 9-10].

[39] Leibniz's 2nd letter to Clarke, no. 1. In Leibniz 1989: 677; cf. *Monadology*, nos. 33-35 (Leibniz 1989: 646), "First truths" (Leibniz 1989: 267-269), *Discourse on Metaphysics*, no. 8 (Leibniz 1989: 307), among many others.

[40] See, e.g., *Monadology*, nos. 31-32.

[41] **Ak.** 4: 270.

[42] **Ak.** 8: 193 et seq.

[43] Kant 1755, section 2.

Indeed, a look through Kant's pre-critical writings shows that in his dogmatic phase he claimed *all* true affirmative propositions to be analytic. In the *Principiorum primorum cognitionis metaphysicae nova dilucidatio*, he announces that there are two principles of all truths, those of identity and contradiction.[44] The point is repeated in Kant's prize essay of 1764, and explained in somewhat greater detail:

> All true judgments must be either affirmative or negative. The *form* of every *affirmation* consists in something being represented as a characteristic mark of a thing, that is to say, as identical with the characteristic mark of a thing. Thus, every affirmative judgement is true if the predicate is *identical* with the subject. [...] The proposition, therefore, which expresses the essence of every affirmation and which accordingly contains the supreme formula of all affirmative judgments, runs as follows: to every subject there belongs a predicate which is identical with it. This is the *law of identity*. The proposition which expresses the essence of all negation is this: to no subject does there belong a predicate which contradicts it. This proposition is the *law of contradiction*, which is thus the fundamental formula of all negative judgements. These two principles together constitute the supreme universal principles, in the formal sense of the term, of human reason in its entirety.[45]

Kant claims, that is, that all true affirmative judgments are identical. A judgment such as

> Germans are Europeans

for example, would be true because its subject (German) is identical to its predicate (European).

Kant's view is, if not defensible, at least more understandable in the light of some of his further assumptions. To begin with, let us note that Kant seems at least sometimes to have thought that a presentation, while serving as predicate in a judgment, does not have its full extension, but rather a more restricted extension determined by the subject of the judgment.[46] In a reflection of 1769, he explains:

> By means of a predicate I do not represent a part of the thing or have a concept of the part, but rather I represent the object itself and have a partial concept of it [....][47]

That is, the predicate represents neither a characteristic belonging to the subject nor the set of things having that characteristic, but rather only those things with the characteristic that are represented by the subject. That (affirmative) judgments are identical then follows directly:

[44] **Ak.** 1: 389; Kant 1992b: 7.
[45] **Ak.** 2: 294; Kant 1992b: 267-268.
[46] The following remarks owe a great deal to a paper by R. George (2004: esp. 327 ff).
[47] Refl. 3921, 1769 [**Ak.** 17: 346]; Kant 2005: 95.

> [I]n every judgment, two concepts apply to one thing. *The thing which I think through the concept A, that very thing I also think through the concept B*—is a judgment.[48]

The claim that the subjects and predicates of true affirmative judgments are identical may also have been helped along by a persistent confusion between presentations (concepts, ideas) and their objects.[49] In the reflection, Kant says that the thing that is *represented by* A is identical to the thing *represented by* B (in the context of the present judgment). But in the Prize Essay, it is the subject and predicate, rather than what they represent, which are said to be identical.[50]

The identity thesis by itself is not decisive proof that the young Kant held all true affirmative judgments to be identical. For it is at least conceivable that there could be identity between what the subject and the predicate (on this occasion) represent even if the predicate is not a constituent part of the subject. In one of his later reflections Kant indeed spoke of identity in connection with synthetic judgments: "In synthetic judgments I represent to myself an identity for the sake of the predicate, not however, the predicate for the sake of the identity."[51]

Nonetheless, his further remarks in the Prize Essay strongly favour the interpretation given above. In the third reflection, Kant notes that Crusius has claimed that the principle of contradiction is not in fact the "supreme and universal principle of all cognition."[52] Crusius had pointed to the difference between the proposition "Every effect has a cause" (which can be demonstrated using the law of contradiction alone) and "Every thing that comes to be has a cause" (which cannot).[53] He remarks: "[. . .] by an effect, one understands something that is brought about by something else, which is called its cause; thus the concept of a cause is already incorporated in the concept of an effect [. . . .]"[54] "But how do I know which things are effects?" he asks.

> Suppose someone says that the thing **A** came to be without any cause; he thereby says something absurd, but not contradictory. For he says that at a certain moment the thing did not yet exist, but now it does. This is no contradiction. For he speaks of different moments. Furthermore, he says that the thing **A** exists, but that another,

[48] Refl. 3933, 1769 [**Ak.** 17: 353 ff.]

[49] One might also argue that profound insight rather than confusion is in evidence here.

[50] Evidence of such a confusion may be found at various places throughout Kant's writings. See, e.g., Kant 1762 (**Ak.** 2: 47; Kant 1992b: 89): "To compare something as a characteristic mark with a thing is to *judge*. The thing itself is the subject; the characteristic mark is the predicate." Also, Jäsche Logic (**Ak.** 9: 58; Kant 1992a: 564): "A *mark is that in a thing which constitutes a part of the cognition of it,* or—what is the same—a *partial representation, insofar as it is considered as ground of cognition of the whole representation.*"

[51] Refl. 3716, **Ak.** 17: 259; Kant 2005: 82.

[52] **Ak.** 2: 293; Kant 1992b: 267. So far, Kant agreed with him, since at that time, as we have seen, he maintained that there were two principles, those of contradiction and identity.

[53] Crusius 1747: §260.

[54] Crusius 1747: 470.

> different thing, **B**, does not. But this is no contradiction. For he speaks of different things. At the same time he says something here that we must think to be completely false and impossible.[55]

It follows, says Crusius, that the principle of sufficient reason cannot be proved from the law of contradiction. §259 contains a list of such propositions, including: "Every force is in a subject, [. . . ,] every substance is someplace, everything that exists is somewhen, [. . . ,] two material objects cannot be in exactly the same place at the same time, [. . . ,] one and the same point of a body cannot be green and red at the same time."[56] Crusius gives us the following account of our knowledge of such propositions.

> Concepts can be constituted either arbitrarily [. . .] or [. . .] as a result of *the nature of the understanding*, namely, in that that is combined which sensation represents as connected, or which we are compelled to think of as connected in that way, so that the concept of the former disappears, or is opposed to our entire nature, if we omit the concept of the latter, and in positing the former wish to deny the latter. [. . .] In this way there arise propositions which are not identical and which ultimately make up the positive [part], or core [*Kernichte*] of our cognition.[57]

In retrospect, it seems fair to say that Crusius here anticipates Kant's analytic-synthetic distinction.[58] But in the Prize Essay, Kant seems to disagree with what Crusius has said, denying that there are any synthetic propositions. There, he divides propositions into the demonstrable and the indemonstrable. The former are proved by analysis:

> The proposition, a body is divisible, is demonstrable, for the identity of the subject and predicate can be shown by analysis and therefore indirectly: a body is *compound*, but what is compound is *divisible*, so a *body* is divisible.[59]

The latter are characterized as follows:

> Any proposition [. . .] is indemonstrable if it is immediately thought under one of these two supreme principles [*sc.*, of identity and contradiction] and if it cannot be thought any other way. In other words, any proposition is indemonstrable if either the identity of the contradiction is to be found immediately *in the concepts*, and if the

[55] Crusius 1747: 471–472.
[56] Crusius 1747: 469.
[57] Crusius 1747: §259, p. 467-8.
[58] Kant would later contest this (**Ak.** 8: 245-6).
[59] **Ak.** 2: 294; Kant 1992b: 268.

> identity and the contradiction cannot or may not be understood through analysis by means of intermediate characteristic marks.[60]

He adds by way of example:

> *[A] body is compound* is an indemonstrable proposition, for the predicate can only be thought as an immediate and primary characteristic mark *in the concept* of body.[61]

A year or so earlier, in his paper on the false subtlety of the four syllogistic figures (1762), Kant had developed a different line of argument for the claim that all true affirmative judgments are analytic. There he claimed that:

> A concept under which other concepts are subsumed is always abstracted, as a characteristic mark, from these subordinate concepts.[62]

If this is so, however, then whenever a concept A is *subordinate* to another concept B (that is, whenever the proposition "All A are B" is true), B must be a *part* of A. From here, it is a short trip to the conclusion that all true (universal) affirmative judgments are (semi-)identical:

> All affirmative judgements are subsumed under a common formula, the law of agreement: *cuilibet subjecto competit praedicatum ipsi identicum* [to any subject whatever there belongs a predicate which is identical to the subject itself].[63]

In arguing for the existence of synthetic judgments in mathematics, then, Kant was at odds with the bulk of respectable German philosophical opinion. More significantly, perhaps, in arguing for the existence of synthetic judgments at all he was in acute conflict with his younger, dogmatic self. Far from speaking metaphorically in framing his definition of analytic judgments in terms of containment, he was insisting that such talk be taken literally. For only when it was so taken was there anything remotely like an explanation of *a priori* knowledge, e.g., as one finds in the following passage of Mendelsohn's:

> [T]here is no doubt that all the truths of geometry are *enfolded* [*eingewickelt*] within the concept of extension, and geometry teaches us how to *unfold* [*entwickeln*] them. For what can the most profound inferences do except dissect a concept, and make distinct what was formerly obscure? They cannot bring forth anything not already to be found within the concept [...] The analysis of concepts is for the understanding

[60] **Ak.** 2: 294; Kant 1992b: 268, emphasis added. Kant's remarks here also shed some light on the definition of analyticity given in the Jäsche Logic (quoted above, p. 2).

[61] **Ak.** 2: 295; Kant 1992b: 268. Emphasis added.

[62] **Ak.** 2: 49; Kant 1992b: 91; cf. **Ak.** 24/2: 568.

[63] **Ak.** 2: 60; Kant 1992b: 104.

> no more than what a magnifying glass is for sight. It does not create anything that is not to be met with in the object; rather, it magnifies the parts of the object, and enables our senses to distinguish a great many things that formerly went unnoticed. The analysis of concepts is no different: it makes the parts and elements of these concepts which were formerly obscure and unnoticed distinct and recognizable, but does not bring forth anything that was not already present in the concept.[64]

Such explanations, feeble as they are in the case of analytic judgments, are completely useless for synthetic ones. This was Kant's great discovery, showing the nullity of much of what he had been taught, and we can certainly forgive him for getting excited about it. But we deprive him of even this modest glory if we seek to improve his definition of analyticity by declaring it to be metaphorical.

*

In modern discussions of the content of judgments (or statements, propositions, etc.), the theory of reference is generally at the centre of things. Insofar as philosophers are inclined to speak of concepts or representations as terms of judgments, these are conceived primarily as determiners of reference (or extensions). The source from which many early analytic philosophers took this approach was Frege, though it is already clearly present in Bolzano's works. But it was uncommon, if not altogether unknown, in Kant's day. Then, the marked tendency was to conceive of representations as ersatz things or, for those who shared Berkeley's views, the things themselves. Representations or ideas, viewed in this way, tend to resemble their objects—as images of the things, objects isomorphic to the things,[65] or as complexes of (representations of) their characteristics.

Viewed against this background, the views of the young Kant and other eighteenth-century philosophers may seem far less strange. If a representation is the isomorphic image of its object in some sense or another, then inspecting it appropriately will reveal the features of that object. Analyzing a representation, becoming distinctly aware of what we already think, though in an obscure and confused way, could, if only we were acute enough, teach us everything there is to know about its object.[66]

[64] Mendelsohn 1838: 294-295.

[65] A view that Leibniz seems to have held at one point. See "What is an Idea?" (1678); Leibniz 1989: 207-208.

[66] Given the widely shared view that so-called *given* concepts often possess hidden complexity, the view that all judgments are analytic might well have seemed irrefutable. For one could always claim that an alleged synthetic judgment was in fact analytic, and that more detailed analysis would show this. See, e.g., **Ak.** 24/1: 153 (Kant 1992a: 121): "Since the concepts in philosophy are already given, [...] the philosopher cannot so easily be certain that he has touched upon all the marks that

Many of Kant's observations reflect this view of things. In his logic lectures, he tells us that microscopes and telescopes can help us to render our representations more distinct.[67] Also:

> If we always knew what we know, namely, in the use of certain words and concepts that are so subtle in application, we would be astonished at the wealth of our cognitions.[68]

Finally,

> In our soul all cognitions actually lie hidden, and all that is necessary is just to develop these cognitions, and bring them into a brighter light.[69]

Thus it is perhaps not so absurd after all for Kant to have regarded his discovery of synthetic judgments as a significant one, especially since it did not involve an abandonment of this conception of representation. For even in his critical phase, Kant's intuitions, at least, remain very much thing-like. What has changed are his views on how these representations come to be. Concepts continue to play a role, to be sure, but other processes are now thought to be involved in bringing unity to a plurality of sensations; thus the characteristics of the objects of experience that the mind fashions cannot simply be read off from the concepts used in their construction.

On the other hand, it is not at all obvious that Kant's innovation, seen in this context, is in any way relevant to modern discussions of content and meaning.

*

Earlier, Crusius was mentioned as one philosopher who seems to have anticipated Kant's distinction between analytic and synthetic propositions. Another is mentioned by Kant himself, namely, Locke,[70] but his reference gives rise to another set of interpretive puzzles. Kant refers to Book IV, chapter iii, §9 of the *Essay*, where Locke discusses "the *Agreement, or Disagreement* of our *Ideas in Co-Existence*." Strangely, Kant does *not* refer to the part of Locke's *Essay*, just a few pages later, where one finds a much more obvious anticipation of his analytic/synthetic distinction, namely, Book IV, chapter viii, on so-called "trifling

belong to a thing, and that he has insight into these completely perfectly. Instead, many *notae* may still belong to the thing, of which he knows nothing." Also, **Ak.** 24/1: 915; Kant 1992a: 357-8.

[67] **Ak.** 24/1: 41; Kant 1992a: 28.

[68] **Ak.** 24/2: 841-3; Kant 1992a: 296-98.

[69] **Ak.** 24/1:123; Kant 1992a: 96. Cf. Leibniz 1903: 10: "Every mind is omniscient, but confused."

[70] Kant 1783: §3 [**Ak.** 4: 270]; cf. his response to Eberhard, **Ak.** 8: 245.

propositions".[71] Locke gives this name to universal propositions which, while certainly true, "add no Light to our Understandings, bring no increase to our Knowledge."[72] Among these, to begin with, he counts all purely identical propositions.

> These obviously, and at first blush, appear to contain no information in them. [...] at this rate, any very ignorant Person, who can but make a Proposition, and knows what he means when he says, *Ay*, or *No*, may make a million of Propositions, of whose truth he may be infallibly certain, and yet not know one thing in the world thereby; *v.g.*, what is a Soul is a Soul [....] What more is this than trifling with Words?[73]

"Alike trifling it is," he continues,

> *to predicate any other part of the Definition of the Term defined,* or to affirm any one of the simple *Ideas* that goes to the making of a complex one, of the Name of the whole complex *Idea*; as *All Gold is fusible*. For *Fusibility* being one of the simple *Ideas* that goes to the making up the complex one the sound *Gold* stands for, what can it be but playing with Sounds, to affirm that of the name *Gold*, which is comprehended in its received signification?[74]

Clearly, Locke was not one to think that all true propositions are analytic in Kant's sense. Nor did he think that all mathematical propositions were analytic, as the following remarks indicate:

> We can know then the Truth of two sorts of Propositions, with perfect *certainty*; the one is, of those trifling Propositions, which have a certainty in them, but 'tis but a *verbal Certainty*, but not instructive. And, secondly, we can know the Truth, and so may be *certain* in Propositions, which affirm something of another, which is a necessary consequence of its precise complex *Idea*, but not contained in it. As that *the external Angle of all Triangles, is bigger than either of the two opposite internal Angles*; which relation of the outward Angle, to either of the opposite internal Angles, making no part of the complex *Idea*, signified by the Name *Triangle*, this is a real Truth, and conveys with it instructive *real Knowledge*.[75]

What conclusions should be drawn from this evidence? First, Kant's contentions that no previous philosopher had recognised the analytic-synthetic distinction[76] and that all previous philosophers held all mathematical truths to be analytic,

[71] It is tempting to think that the *Prolegomena* contain a misprint, and that Kant meant to refer to Book IV, chapter viii instead of Book IV, chapter iii; but his paraphrases indicate that he did indeed mean the latter.

[72] Locke 1975: IV, viii, §1. Cf. Frege 1884:101; 1892: 25.

[73] Locke 1975: IV, viii, §3.

[74] Locke 1975: IV, viii, §5.

[75] Locke 1975: IV, viii, §8.

[76] We have already seen that Crusius seems to have done so.

are simply false. Second, Kant, like many other philosophers, may not have had the stamina to read through Locke's *Essay* right to the end, at least not while in an attentive frame of mind. This should come as no surprise. Kant (as he himself would have been the first to admit) was no historian, and he is by no means the first philosopher to mistake what he knew of the development of philosophy for the history of the subject.

Locke is interesting for other reasons as well. His distinction between trifling and non-trifling propositions is, unlike Kant's analytic-synthetic distinction, explicitly framed in linguistic terms. Trifling propositions, one may fairly say, are true in virtue of meaning. Indeed, Locke comes close to affirming the converse, maintaining that there are but three kinds of trifling propositions, namely, identical propositions (e.g., "Oyster is Oyster"), semi-identical propositions (e.g., "Red apples are red" or "Gold is yellow", where "Gold" means "a yellow metal, etc."), and propositions "wherein two abstract Terms are affirmed one of another", for example, "Parsimony is Frugality."[77] Locke also explicitly states that trifling propositions do not increase our knowledge, a claim we have seen some reason to doubt Kant ever made with respect to analytic judgments. In many ways, it seems to me, Locke is a better candidate than Kant for the source of what later came to be called the analytic/synthetic distinction.

Bolzano on analyticity

Bolzano first mentions the analytic/synthetic distinction in the first and only published installment of the *Beyträge zu einer begründetern Darstellung der Mathematik*,[78] but there he seems to be in full agreement with Locke. Analytic judgments, defined as those of the form 'A which is B is B', he finds to be so utterly trivial that they do not even deserve to be called judgments:

> [Analytic judgments] do not deserve the name *judgment*, rather only that of *proposition* [*Satz*], in that they teach us nothing new as judgments, but only in so far as they are expressed in words; or, in other words, that the new information which can perhaps be obtained from them concerns not the concepts or the things as such, but at most only their designations.[79]

[77] Locke 1975: IV, viii, §§12-13.

[78] Bolzano 1810: II, §17.

[79] Bolzano 1810: II, §18. Why does he say this? Perhaps he has in mind sentences such as 'A circle is a plane figure.' (It seems clear that here Bolzano uses '*Satz*' for the verbal expression of a judgment.) Here, the term 'circle' designates a complex concept which might be more distinctly expressed as 'plane figure, composed of all the points lying at a given distance from a given point'. In the *sentence* 'A circle is a plane figure', the triviality of the judgment expressed is not immediately

Later, Bolzano came to see more value in Kant's contribution, first because it drew attention to another important distinction, and second, because Kant's analytic propositions were a special case of a genuinely interesting logical phenomenon.

On several occasions in the *Theory of Science*, Bolzano expresses his gratitude to Kant for having drawn attention to the distinction between analytic and synthetic judgments. The importance of Kant's distinction, as Bolzano sees it, is that it forces us to distinguish between the attributes of an object and the parts of an idea representing the object.[80] The term '*Merkmal*' (mark, characteristic, note) was used by many philosophers at that time in both of these senses,[81] and the equivocation gave rise to all sorts of further confusions. One of these was embodied in the claim, examined by Bolzano in §64 of the *Theory of Science*, that if $[A]$ is an idea[82] of an object α, then for every attribute P of α there is a corresponding part of $[A]$ that represents P.[83] A further consequence of this assumption is the thesis that the content and extension of ideas vary inversely: the more parts an idea has, the fewer objects it represents and conversely. Despite his discovery of the analytic/synthetic distinction, Kant affirmed the thesis in the Jäsche Logic.[84] Bolzano refuted it with a counterexample ([A man who speaks all living European languages] has a greater content and a greater extension than [A man who speaks

evident. If we are told that this sentence expresses an analytic judgment, however, it does tell us something about the content of the concept designated by the word 'circle'. On this account, granted, Bolzano's formulation would be defective, since, in order to obtain this information, we need to be told that the sentence expresses an analytic judgment, rather than simply a true one. But it is already clear that his statement is at best elliptical, since he says that analytic judgments may teach us something new only insofar as they are expressed in words, from which one gathers that were they not expressed in words, they would not be informative. But then, clearly, the judgment in and of itself is *not* teaching us anything new.

[80] 'Idea' here translates '*Vorstellung*', as is common practice in translations of Bolzano's works. This term is usually rendered as 'presentation' or 'representation' in translations from Kant's writings.

[81] Including Kant himself. See the Jäsche Logic, VIII (**Ak.** 9: 58; Kant 1992a: 564); quoted above, note 50, p. 12.

[82] It is common in the literature on Bolzano to place square brackets around a sentence to form a designation of the proposition in itself it expresses, and I shall follow this convention, applying it not only to sentences but also to other similar entities. Single quotes are used to indicate mention of linguistic expressions. For example, '$[2 \cdot 2 = 4]$' will designate the proposition in itself expressed by '$2 \cdot 2 = 4$', while '[2]' designates the idea in itself designated by '2'. These conventions are not applied to quotations from Bolzano's works.

[83] Or, in general, that if an idea $[A]$ represents a set of objects S, then for every attribute P that (necessarily) belongs to all the members of S, there is a corresponding part of $[A]$ that represents P. §63 examines the related claim that the parts of an idea are in one-to-one correspondence with the parts of its object.

[84] See §7 [**Ak.** 9: 95; Kant 1992a: 593]; noted by Bolzano (**WL**, §65 [I.292-3]).

all European languages]) and thanked Kant nonetheless:

> If I have been fortunate enough to avoid an error that others unwittingly fell into, I freely admit that I owe this to KANT, whose distinction between analytic and synthetic judgments would not exist if all attributes of an object had to be constituents of its idea.[85]

Kant's reminder thus seems to have been one of the things that encouraged Bolzano to take a closer look at the theory of ideas, an inquiry that resulted in a radically different approach to this part of logic. In most of the theories Bolzano encountered, ideas were thought to play two quite different roles. On the one hand, they were supposed to serve as the terms of judgments (subjects and predicates). On the other hand, they were often taken to be mental copies of their objects, resembling them (in some way or other) right down to the minutest details. Often, visual language was used when describing ideas. Kant, for instance, said that an intuition (a kind of idea, or representation) of a house contained intuitions of the doors, windows, etc., as parts.[86] Indeed, he had gone so far as to say that "what we call external objects are nothing but mere presentations [*Vorstellungen*] of our sensibility."[87]

Bolzano could never understand how one and the same entity could play both these roles. Since he thought that logic was primarily concerned with the laws of truth,[88] and that truth was a property of propositions, he took the first feature to be primary, and simply discarded the second as incompatible with it. Ideas in his sense are simply sub-propositional parts of propositions. As is well known, he distinguishes between subjective ideas and their objective counterparts (ideas in themselves); both are distinguished from linguistic signs, and Bolzano provides a detailed theory of the relations between signs and ideas (and propositions), including a surprisingly nuanced theory of interpretation.[89] He expressly denies that ideas are copies of their objects, a second set of objects precisely resembling the things outside the mind; rather, they *have* objects:

> [T]he idea of an object cannot, strictly speaking, be called its picture. It is not an object which we examine in place of another; rather, it is what arises in our mind when we examine this object itself.[90]

Bolzano's radicalism on this point becomes clear when he speaks of the individuation of ideas. With respect to ideas with objects, he recognizes only three

[85] **WL**, §120 [I.571]. For further discussion of these matters, see Künne 2001.
[86] Kant 1800: Einleitung, V. [**Ak.** 9: 34].
[87] Kant 1781/1787: A30/B45.
[88] **WL**, §16.3.
[89] **WL**, §§285, 385-387; see also §§637 ff.
[90] **WL**, §52.5 [I.231].

individuating attributes: extension, content (the sum of the parts of a complex idea), and structure (the way the parts of a complex idea are combined, their *Verbindungsart*).[91] In the case of simple ideas with objects, where there is no content strictly speaking and hence no structure, ideas are individuated solely by their extensions. If asked why a certain simple idea has the extension it does, Bolzano would respond that the relation between idea and its object(s) is a primitive one and hence no explanation in terms of more basic relations is to be had. In particular, any sort of attempt to explain why an idea has or represents the objects it does in terms of resemblance is rejected in the strongest terms.[92]

It is tempting to to employ Kuhnian language here, for the theories of ideas of Bolzano and Kant are so far apart that it seems quite reasonable to say that they inhabited different worlds. One thing is certain: Bolzano could not have thought the analytic-synthetic distinction important for the same reasons Kant did. Having rejected those features of traditional theories of ideas that might have made it seem even somewhat plausible to claim that all true affirmative judgments are analytic, Bolzano must have had difficulty in seeing any importance at all in Kant's discovery of synthetic judgments.

In the *Theory of Science*, he offers several criticisms of Kant's definitions. The first of these we have already seen: Analytic judgments (or propositions) in Kant's sense are trivial:

> I believe that no merely analytic truth deserves a place in a science, be it logic or some other. For such analytic truths are in my opinion too unimportant to merit presentation as one of the propositions of a science. Who would want, for example, to fill up geometry with propositions like: An equilateral triangle is a triangle, or an equilateral figure, etc.?[93]

In §148, he adds a couple of other minor criticisms.[94] But his final observation is the kicker. Kant had rightly sensed, he thought, that there was something genuinely important about what he had called analytic judgments. Yet he had failed to put his finger on just what it was. For Bolzano, containment of the predicate in the

[91] **WL**, §§56, 66, 96.

[92] Cf. Bolzano 1843: 183; **BBGA** I.18: 74-75.

[93] **WL**, I §12 [I.51-52]. Later in the **WL**, Bolzano maintains that some analytic propositions do deserve to be included in scientific treatises (§447). Indeed, he maintains that many important theorems of mathematics are analytic (§305 [III.241]). There is no reason to find a contradiction in his remarks. In §12, not yet having given his own definition, he clearly uses 'analytic' in Kant's sense, and once again agrees with Locke's view of (Kant-)analytic propositions as trivial. Later, after he has introduced his own definition of analyticity (§§147-148), Bolzano states that many (Bolzano-) analytic propositions are in fact important and anything but trivial. On this point, cf. Künne 2006: 235, note 49.

[94] These are discussed in detail in Siebel 2011.

subject is just a distraction. While one may rightly say, with some qualifications, that a proposition like

> [A red apple is an apple.]

is true because its predicate is contained in its subject, Bolzano thought it far more significant that the ideas [red] and [apple] are inessential to its truth. That is, they occur vacuously, in the sense that the truth value of the proposition would not change if they were replaced by other ideas such as [unmarried] and [man]:

> I believe that th[e] importance [of analytic propositions] lies in the fact that their truth or falsity does not depend upon the individual ideas of which they are composed, but that it remains the same irrespective of the changes to which some of their ideas are subjected [. . . .][95]

The truly interesting feature of Kant's analytic propositions, that is, is the invariance of their truth-value under an entire class of transformations. Bolzano now proposes that we latch onto this feature alone, abandoning the rest of Kant's account, and without further ado requisitions Kant's term to designate the new, radically different concept.

Bolzano's definitions

Central to Bolzano's redefinition is his conception of propositional form. Every proposition has parts, and the structure of propositions is such that where a given part occurs, others may be put in its place, without the result ceasing to be a proposition. By considering certain parts of a proposition candidates for such substitutions, we obtain a class of propositions generated by substitution or variation. For example, if in a simple categorical proposition such as:

> [Aristotle is wise.]

we consider [Aristotle] variable, we obtain a class of propositions including:

> [Socrates is wise.]
>
> [Bush is wise.]
>
> [Chomsky is wise.]
>
> etc.

while if both [Aristotle] and [wise] are tagged as variables, we may also obtain propositions such as:

[95] **WL**, §148 [II.88].

[Leibniz is clever.]

[Caligula is vicious.]

etc.

Associated with these classes of propositions are linguistic expressions containing letters to mark places for variables. In the above cases, for example, these would be:

- X is wise.
- X is y.

Either the class or the linguistic expression might be called a propositional form, according to Bolzano, though he thinks that, strictly speaking, only the linguistic expression should be.[96]

Perhaps the most important thing to note about Bolzano's conception of propositional form, and indeed logical form in general, is that there is no *unique* form for any given proposition (or set of propositions, argument, etc.). Rather, by considering now these, now those, parts of a proposition variable, we obtain different forms. This flexible conception of form is particularly valuable in mathematics, where it is routine to consider one and the same proposition as belonging to a variety of different forms. An equation as simple as:

$$[(1+1)=(1+1)]$$

for example, might be considered under any of the following forms:

- $a = b$
- $a = a$
- $(a + a) = (a + a)$
- $(a + b) = (a + b)$
- $(a + b) = (b + a)$
- $(a * a) = (a * a)$
- $(a * b) = (a * b)$
- $(a * b) = (b * a)$
- $(a * b)R(a * b)$
- aRa
- aRb, etc.

Because form depends upon a specification of variable parts, there is no sense in speaking of *the* form of a proposition.[97] Rather, form and matter are relative

[96] **WL**, §12 [I.48]; cf. §81, note 2.

[97] This remark may seem to be in tension with Bolzano's claim that all propositions are contained under the form 'A has b' (**WL**, §§81, note 2; §127). It is not. For a proposition's standing under one form, as we have just seen, by no means precludes it from standing under others.

terms: what counts as formal is fixed by what we happen to be interested in within a given context.[98] Because there is no unique form for any given proposition, it does not make sense to say simply that a proposition is formally true. Rather, one must say that it is formally true relative to a specified form (or, as Bolzano puts it, with respect to specified variable ideas).

In §147 of the *Theory of Science*, Bolzano defines the concepts of *comparative* and *universal validity*, as well as those of *comparative* and *universal invalidity*. Often, when we make substitutions for some parts of a given proposition, we obtain both true and false results. Beginning, for example, with a proposition such as:

[The human being Solon is wise.]

and treating [Solon] as the only variable part, we will find that some substitutions produce true propositions, others false ones.

Suppose now that we have a set of ideas which satisfies the following conditions: a) for each human being, there is an idea in the set that represents it and it alone; b) the only ideas in the set are those that represent individual human beings; and c) no two ideas in the set represent the same human being. The ideas belonging to this set can then be mapped one to one onto the set of human beings, and the ratio of the number of ideas in this set which produce true propositions when substituted for [Solon] to the total number of ideas in the set will accordingly be equal to the probability that a randomly selected human being is wise. Bolzano calls this the *degree of validity* of a proposition with respect to given variable parts.[99]

If, by contrast, we begin with the proposition:

[The human being Solon is perfect.]

and again tag [Solon] as the only variable, we find that the degree of validity of the proposition is zero (all substitutions result in false propositions). This limiting case of relative validity Bolzano calls *universal invalidity* relative to given variables.

Defining universal validity is a little more complicated, due to the fact that Bolzano, in line with logical tradition, held propositions with empty subject-ideas to be false. With this view, even a proposition such as:

[The human being Solon is either wise or not wise.]

would have false variants, e.g.:

[The human being $\frac{\pi}{6}$ is wise or not wise.]

[98] Cf **WL**, §254 [II.516]: "In my opinion, what belongs to the *form* of a logical object are those of its attributes which determine the kind of object the logician considers it to be. Everything else which belongs to it only accidentally belongs to its *matter*"

[99] **WL**, §147.

With this in mind, Bolzano frames his definition of universal validity as follows:

> Let the proposition A be such that all the propositions which can be generated from it are true, if the ideas $i, j, \ldots$ alone are considered variable, and if only objectual propositions may be formed. Then the degree of validity of A with respect to $i, j, \ldots$ is the largest possible, i.e., it equals 1, and we can call the proposition *universally* or *fully valid*.[100]

(where by 'objectual proposition', Bolzano means a proposition whose subject-concept has an object).[101]

Analytic propositions, finally, are defined as those which are either universally valid or universally invalid relative to *some* specification of variable parts; the remaining propositions are called *synthetic*.[102] Note that because his definition embraces universally *invalid* propositions, there are false as well as true Bolzano-analytic propositions. Thus Bolzano's analytic-synthetic distinction, unlike Kant's, covers all propositions, not merely the true ones.

Bolzano-analyticity is, fundamentally, a notion of invariance (namely, of truth-value) under some class of transformations. That he does not, like later authors such as Quine and Tarski, confine his attention to cases where the transformations affect all and only the non-logical parts of propositions may be attributed to his habitual striving for maximal generality. From his point of view, Quine's logical truth would appear as just one of many special cases of the more general phenomenon singled out by his definition.

Here are some of Bolzano's examples of analytic propositions, true as well as false, where the underlining or the letters indicate the variable parts:

1. [The man Caius is mortal.] (**WL**, §147)

2. [The man Caius is omniscient.] (**WL**, §147)

[100] **WL**, §147 [II.82].

[101] More generally, Bolzano (**WL**, §147 [II.80]) expresses these constraints as follows: the substitutions must produce subject-ideas that are *objectual* [*gegenständlich*], i.e., which have an object, and no two of the resulting subject-ideas may be *equivalent*, i.e., represent the same object(s). Other restrictions on substitution have to do with what we might, following Husserl, call *semantic category*. The thought here is that meanings and partial meanings naturally fall into different kinds, and that in substitution an idea of a given kind should always be replaced by another idea of the same kind (for example, [Socrates] should never be replaced by [between] or [and]). If the boundaries between these kinds are not respected, the result is often something that does not seem to be a meaning at all, just as the violation of grammatical categories in substituting expressions for each other in sentences can result in nonsense. On this point, see Künne 2006 and Siebel 1996: 73ff.

[102] **WL**, §148.

3. [A is A.] (**WL**, §148)
4. [An A which is a B is an A.] (**WL**, §148)
5. [An A which is a B is a B.] (**WL**, §148)
6. [Every object is either B or non-B.] (**WL**, §148)
7. [If all men are mortal and Caius is a man, then Caius is mortal.] (**WL**, §315)
8. [If A is larger than B, then B is smaller than A.] (**WL**, §148)
9. [If $P = Mm$, then $M = \frac{P}{m}$.] (**WL**, §148)
10. [The soul of Socrates has been annihilated.] (**WL**, §369)
11. [The soul of Socrates is a simple substance.] (**WL**, §447)
12. [If $\frac{a^2}{2} = b$, then $a = \pm\sqrt{2b}$] (**WL**, §447)

I will take this opportunity to address a point of detail. It is often assumed that Bolzano only allowed for *exhaustive* substitutions, in the sense that if an idea occurs repeatedly in a proposition and is declared variable, all occurrences must vary (and uniformly, at that).[103] For example, it is thought that on Bolzano's account, from a proposition such as:

[Cato killed Cato.]

we might obtain, via substitution for [Cato],

[Caesar killed Caesar.]

But not:

[Caesar killed Cato.]

On this interpretation, Bolzano's understanding of substitution lines up with Tarksi's, as seen, for instance, in "On the Concept of Logical Consequence."[104] A different approach to substitution (uniform, but not necessarily exhaustive), may be found in Frege's *Begriffschrift*:

[103] See, e.g., Bar-Hillel 1952: 310; Berg 1987: 16; Siebel 1996: 64-65. Wolfgang Künne has pointed out a certain difficulty with the claim that one and the same idea in itself can occur repeatedly in a proposition. See his discussion of the so-called "repetition problem" in Künne 1997: 223 ff; see also Künne 2001: 278–280.

[104] Tarski 1983: 415.

> If, in an expression (whose content need not be assertible), a simple or a complex symbol occurs in one or more places and we imagine it as replaceable by another (but the same one each time) at all or some of these places, then we call the part of the expression that shows itself invariant a function and the replaceable part the argument.[105]

Mark Siebel has a rather ingenious argument (which I do not have the space to discuss here) in favour of the thesis that, given Bolzano's theoretical commitments, all substitutions must be both uniform and exhaustive.[106] I am not sure why other commentators have thought this. I can say, however, that Bolzano's examples include cases where (in the Fregean manner) only some occurrences of a given idea are varied. The most obvious of these may be found in §447 of the *Theory of Science*, where Bolzano declares the proposition:

[If $\frac{a^2}{2} = b$, then $a = \pm\sqrt{2b}$.]

to be universally valid with respect to the variable idea [2], and soon afterwards makes it clear that only some occurrences of this idea are to be considered variable, namely, those indicated by the underlining below:

[If $\frac{a^2}{\underline{2}} = b$, then $a = \pm\sqrt{(\underline{2}b)}$.]

Thus even if Siebel's argument is sound, it would show at most that Bolzano's practice was inconsistent with his theory, not that he never contemplated non-exhaustive substitutions. Clearly, he did.[107]

This feature makes a difference. Consider, for example, the propositions:

$$[\sin^2(1) + \cos^2(1) = 1]$$
$$[\sin^2(2) + \cos^2(2) = 1]$$
$$[\sin^2(3) + \cos^2(3) = 1]$$

If we require substitutions to be both uniform and exhaustive, only the last of these equations could be declared analytically true, since it could be considered to belong to the form:

$$\sin^2(x) + \cos^2(x) = 1$$

If we exhaustively and uniformly replace [1] in the first equation or [2] in the second, however, we obtain the forms:

[105] Frege 1879: §9.

[106] Siebel 1996: 64-65.

[107] Cf. **WL**, §375 [III.475-6], where Bolzano speaks of replacing an idea "everywhere that it occurs"—a qualification that seems to me to indicate that he thought it permissible to replace it at only some of these occurrences.

$$\sin^2(x) + \cos^2(x) = x$$
$$\sin^x(x) + \cos^x(x) = 1$$

And there are certainly false objectual propositions of these forms. It is also clear that no other selection of variables will result in a form encompassing only true propositions (subject to the objectuality constraint). If, by contrast, we permit non-exhaustive substitutions, we can declare all three propositions analytically true.

Logical Analyticity

Bolzano also defines a narrower concept, which he calls *logical* analyticity as opposed to the garden variety, which he calls analyticity in the broader sense.[108] The distinction is introduced with the help of the following examples:

> [A depraved man does not deserve respect.] (universally valid w.r.t. [man])
>
> [A man may be depraved and still enjoy continued happiness.] (universally invalid w.r.t. [man])
>
> [An A which is a B is an A.] (universally valid w.r.t. $[A]$, $[B]$, both occurrences of $[A]$ varied uniformly)
>
> [Every object is either B or non-B.] (universally valid w.r.t. $[B]$, both occurrences varied uniformly)

He writes that the difference between the first two and the last two cases

> lies in the following: In order to appraise the analytic nature of the [latter], nothing but logical knowledge is necessary, since the concepts which form the invariable part of these propositions all belong to logic. On the other hand, for the appraisal of the truth and falsity of propositions like [the former] a wholly different kind of knowledge is required, since concepts alien to logic intrude.[109]

The key phrase here is the following: "the concepts which form the invariable part of these propositions all belong to logic." That is, *only* logical concepts remain invariant. Though this statement seems clear enough, some commentators have appended the converse, and Bolzano has accordingly been taken to mean by this that *all and only* logical concepts remain invariable.[110] Accordingly, logically

[108] **WL**, §148.3 [II.84].

[109] **WL**, §148.3 [II.84]. Cf. the similar remarks of Tarski (1983: 418-419).

[110] See, e.g., Bar-Hillel 1952: 310; Berg 1987: 18. Siebel (1996: 115-116) makes a similar statement about logical deducibility. A notable exception is Morscher 1999.

analytic propositions would be those that are universally valid (or invalid) when precisely the non-logical ideas are considered (uniformly) variable. This interpretation is tempting, in that it would seem to bring Bolzano's notion in line with Quine's concept of logical truth (+ logical falsity) or Tarski's notion of analyticity (+ contradictoriness).[111] Yet this does not seem to be what Bolzano said.

Consider, by way of example, a proposition such as:

[A man, who is rich and powerful, is rich and powerful.]

If we consider uniformly variable the parts indicated by the boxes:

A $\boxed{\text{man}}$, who is $\boxed{\text{rich and powerful}}$, is $\boxed{\text{rich and powerful}}$.

We have a proposition belonging to the following form:

An A, which is B, is B.

and here one may say, with Bolzano, that the invariable concepts all belong to logic, so that the proposition may be called logically analytic. At the same time, since [and] is not held fixed, it is false to say that all and only non-logical concepts are variable.

It might be objected at this point that the original proposition is also universally valid with respect to the finer form:

An A, which is B and C, is B and C.

and one might also wish to argue that whenever a proposition is analytic with respect to a form whose only invariable parts are logical concepts, it is also universally valid with respect to the form in which all and only the logical concepts are invariable. Given this, it might seem that I am merely quibbling.

There are a couple of reasons to think that this is not the case. Consider, for example, the following proposition, which (on at least one fairly widespread understanding of what the logical concepts are) contains only logical concepts:

$$[\exists x \exists y x \neq y \vee \neg \exists x \exists y x \neq y]$$

If we require *all* logical concepts to remain invariant, there would be nothing to vary, in which case the proposition could not be declared logically analytic.[112] If, by contrast, we only require a proposition to be universally valid with respect to

[111] Quine 1976: 110 note; Quine 1960: 65, note 3; Tarski 1983: 417–418. The additions are due to the fact that Bolzano uses 'analytic' to cover universally invalid as well as universally valid propositions.

[112] Cf. Künne 2006: 201. Similar remarks apply to Künne's examples P9† and P11* on p. 200.

a form, the only invariable parts of which are logical concepts, we can declare this proposition to be logically analytic, as it belongs to the form:[113]

$$P \vee \neg P$$

A second, more significant difference between the two proposed interpretations comes to light when we consider Bolzano's well-known remark on the difficulty of establishing a sharp boundary between the logical and the non-logical concepts:

> This distinction, I admit, is somewhat unsettled [*hat sein Schwankendes*], as the whole domain of concepts belonging to logic is not circumscribed to the extent that controversies could not arise at times.[114]

Suppose, like Bolzano and Tarski, we were not entirely sure where to draw the line or even that there is no precise line to be drawn (as seems entirely possible given Bolzano's pragmatic approach to determining the boundaries of sciences[115]). On Bolzano's account, as I understand it, we would still be able to declare some propositions logically analytic with full confidence, provided that we were confident that at least some concepts are rightly classed as logical. For even if a proposition contained many concepts of whose status we were unsure (or which was even indeterminate), we could still confidently pronounce it logically analytic if it was universally valid (invalid) relative to a form whose only invariable parts are indisputably logical concepts.

Analyticity, necessity, and apriority

Analyticity, in Bolzano's understanding, has no special epistemic or metaphysical status—the vacuous occurrence of certain ideas may happen for all sorts of reasons, necessary as well as contingent, known and unknown, etc. In defining analyticity,

[113] It might be objected here that Bolzano's definitions speak only of the variation of *ideas* and make no provision for varying entire *propositions*, as seems to be required in the case of the form '$P \vee \neg P$'. The reply is provided by Bolzano himself. According to him, a proposition expressed by a sentence of the form 'A or B' would be more distinctly expressed as: 'The idea of a true proposition among A, B, has objectuality.' (**WL**, §166; see also Bolzano and Exner 1935: 11 [Bolzano 2004: 94-95]) In this second expression, 'A' and 'B' do not mark places for propositions, but rather for *ideas* of propositions. By varying these ideas, we obtain in effect the same results as we would by varying the propositions. Similar remarks apply to (external) negation, Bolzano maintaining that propositions expressed by sentences of the form 'Not A' are more distinctly expressed as: 'The proposition A has lack-of-truth.' (WL, §189, no. 1e), where 'A' marks a place for an *idea* of the relevant proposition, an element that may thus be varied according to his conception. Thus Bolzano can do some propositional logic after all.

[114] **WL**, §148.3 [II84].

[115] See **WL**, §§409-427.

he was not attempting to explain necessity or apriority, and it should come as no surprise that his conception is of little or no use for such a project.

To begin with, it is obvious that among the Bolzano-analytic propositions, there are not only necessary, but also contingent truths. For example, the contingent proposition:

> (*) [Theodore Roosevelt, a 20th-century US president, was not born in Hawaii.]

which is universally valid with respect to [Theodore Roosevelt], is analytic (recall that the objectuality constraint rules out substitutions of ideas not representing 20th-century US presidents). The same example shows that not all analytic truths can be known *a priori*; it also shows that the *analyticity* of a given proposition may not be recognizable *a priori*. Interestingly, too, the related universal proposition:

> [No 20th-century US president was born in Hawaii.]

from which (*) is deducible, is synthetic.[116]

Bolzano was fully aware of these features of his definition. In §197 of the *Theory of Science*, he notes that there are propositions of each of the following kinds:

- analytic and purely conceptual
- analytic and intuitional
- synthetic and purely conceptual
- synthetic and intuitional[117]

Since he equates necessary truth with purely conceptual truth,[118] it is clear that he recognises that there are both necessary and contingent analytic truths. Moreover, since only purely conceptual truths may be known *a priori*,[119] it is clear that he recognizes analytic truths that can only be known by experience.

[116] Cf. **WL**, §447.

[117] For Bolzano, an intuition is an idea (representation) that, while simple, has exactly one object (**WL**, §72). He maintains that representations of contingently existing particulars always either are intuitions or else contain intuitions as parts (**WL**, §182). Thus, by his lights, any proposition about a contingently existing individual is intuitional. This is so even if the individual is no longer with us, e.g., Socrates. For further discussion of Bolzano's concept of an intuition, see George: 2004; Bolzano 2004: 14-20; Textor 1996: Part I.

[118] **WL**, §182.4; though Bolzano considers this an improper sense of 'necessary', it is what he has by way of a general conception of necessary truth.

[119] See **WL**, §133, note; §306.12, §369 [III.455].

Generally speaking, there is no reliable connection between analyticity and apriority. Things are different, however, in certain special cases. To begin with, let us consider once again Bolzano's notion of logical analyticity. In order to recognise the analyticity of propositions of forms such as 'An A which is a B is an A' or 'Every object is either B or non-B', he says,

> no other than logical knowledge is necessary, since the concepts which form the invariable part of these propositions all belong to logic.[120]

Given that §148 is part of the Theory of Elements, which deals with propositions and ideas in themselves, regardless of whether they are thought or expressed by anyone, it seems reasonable, at least at first blush, to assume that the second characterization is primary, and invoked to explain the first.[121] If this interpretation is correct, we do have a second point of contact with Kant, and perhaps even partial justification for Bolzano's remark that Kant was the first to make appropriate use of the analytic-synthetic distinction.[122] For even with an intuitional proposition such as

[Either Socrates is wise or he isn't.]

a proposition claiming the logical analyticity of this proposition, e.g.:

[Every disjunction of a proposition with its negation is true.]

might well be purely conceptual. And since Bolzano holds that many purely conceptual propositions can be known *a priori*,[123] it seems reasonable to suppose that he would have thought that logical analyticity can be recognised *a priori* in many cases.

As Coffa appears to have recognised, similar remarks apply to other cases of "purely conceptual analyticity", e.g., the analyticity of a true proposition such as:

[This, which is red, is not green.]

[120] **WL**, §148.3 [II.84].

[121] This is Wolgang Künne's interpretation. Mark Textor has argued for the contrary position. See Künne 2006: 203 and 238 note 71, Textor 2000: 448ff.

[122] **WL**, §148, note 4 [II.87].

[123] Interestingly, Bolzano consistently avoids claiming that all purely conceptual propositions can be known *a priori*. In **WL**, §133, for instance, he claims that the distinction between propositions that can and those that cannot be known *a priori* "nearly coincides" with that between purely conceptual and intuitional propositions. He also says that *most* purely conceptual propositions can be known *a priori.* He provides no argument in support of this optimistic estimate.

which is universally valid with respect to the idea [this].[124] If, as Bolzano thought, the ideas [red] and [green] are pure concepts,[125] the proposition would be universally valid relative to a form, the only invariable parts of which are pure concepts. And if we can recognize *a priori* the truth of the purely conceptual proposition that nothing which is red is green, we will by the same token be in a position to recognize *a priori* that the above empirical proposition is analytic if it is true.[126]

Supposing that we are able to recognise conceptual analyticity *a priori* in certain cases, it might seem that we should also able to recognise the *truth* of propositions that are conceptually analytic without the benefit of experience. According to Bolzano, however, this is not always the case, again due to the objectuality constraint. In the above example, for instance, if the idea [this] happened not to represent something which is red, the subject term would have no object, and the proposition would be false.[127] The case is different with universally *in*valid intuitional propositions, however, as Bolzano explains in §369 of the *Theory of Science*:

> If the proposition is *empirical*, i.e., contains intuitions, then we may know in advance that through mere concepts we could at most show its falsity, but never its truth. The former case occurs when closer consideration reveals that the proposition belongs to the special sort of analytic propositions that remain false no matter what ideas we substitute for the intuitions they contain. Of this sort would be, for example, the proposition that the soul of Socrates has been annihilated; for a purely conceptual truth teaches us that no soul is annihilated, so that the above proposition is false no matter what object the intuition "Socrates" refers to.[128]

Thus there is some connection between special cases of analyticity and apriority on Bolzano's conception, and we have another small point of contact between his doctrine and Kant's.

Conclusion

I began with the observation that Kant is frequently viewed as belonging to, and perhaps even initiating, a tradition of reflection on the notion of analyticity, a tra-

[124] Coffa 1992: 35.

[125] At **WL**, §286 [III.88-89], B. says he inclines towards this view.

[126] Cf. **WL**, §§223, 367–368.

[127] On this point, cf. Whitehead and Russell 1957: Vol. I, p. 182: "Thus such a proposition as 'the man who wrote Waverley wrote Waverley' does not embody a logically necessary truth, since it would have been false if Waverley had never been written, or had been written by two men in collaboration. For example, 'the man who squared the circle squared the circle' is a false proposition."

[128] **WL**, §369 [III.454].

dition extending through Frege to the Vienna Circle and beyond. Seen in this light, Bolzano seems obviously out of step in failing to acknowledge what everyone must see as reasonable theses about analyticity. Having developed most of the tools required to give a thoroughly modern account of analyticity, some of his readers have asked, why did he obstinately refuse to do so?

William and Martha Kneale simply raised an eyebrow at Bolzano's concept, noting some of its "curious" features.[129] Yehoshua Bar-Hillel, by contrast, was so completely taken aback by Bolzano's notion of analyticity in the broader sense that he simply assumed that he could not have meant what he said. So secure was he in this conviction that he felt obliged to conjecture a history of the manuscript of the *Wissenschaftslehre* in which Bolzano finally saw the light, but too late to stop the presses.[130] Certain that the only reasonable notion for Bolzano to be aiming at was something like one of Carnap's versions of analyticity, he simply writes off analyticity in the broader sense:

> [The reader] must have become convinced of the almost ridiculous inadequacy of Bolzano's definition of 'analytic'. [...] We can only wonder about the lack of perspective which caused him to believe that his definition is only "somewhat broader", but even in this case, we must ask, what prevented him from looking for a more adequate definition?[131]

Coffa, perhaps also with Carnap in mind, seems to have interpreted Bolzano's reflections on analyticity as an attempt to explain necessity,[132] apparently unaware of the latter's definition of necessity in terms of purely conceptual truth.[133] He furthermore appears to confuse what Bolzano calls subjective grounds (or grounds of knowledge) with objective grounds, and draws the conclusion that Bolzano believed the purely conceptual truths of logic and mathematics to be empirically grounded.[134] Not surprisingly, he deems Bolzano's efforts to account for necessity a failure.

Judgments such as these would no doubt have been modified had those who formed them had the opportunity to become better acquainted with Bolzano's philosophy. (Sadly, Coffa died before his book had been published, still less reviewed.) But even those who have had the time to get to know Bolzano well may

[129] Kneale and Kneale 1962: 366 ff. They had similar misgivings about Bolzano's concept of deducibility. See also Kneale 1961.

[130] Bar-Hillel 1950: section 7.

[131] Bar-Hillel, 1950 40. Elsewhere (p. 42), he claims that Bolzano's expression 'logically analytic' is pleonastic.

[132] Coffa 1992: 33 *et seq.*

[133] **WL**, §182.4.

[134] Coffa 1992: 38.

be forgiven for feeling a certain uneasiness concerning what he says about analyticity. For there is no doubt that his use of the term 'analytic' is completely out of line with what is now well-established usage, something that cannot fail to wrong-foot us despite our best efforts. But we should not be fooled by this into thinking that, for Bolzano, the term 'analytic' had or should have had any of the associations we take for granted. In his day, he was perfectly justified in thinking that the term was used only to designate a decidedly uninteresting concept; judging that it was not required for anything of consequence, he simply appropriated it for another purpose.

This may be thought a tendentious choice of terminology, and perhaps in some respects it is. But it can also be looked upon as a polite way of conveying his opinion that much of what Kant and his followers had to say about analyticity was mistaken or at least misguided, while at the same time acknowledging that Kant had, here as elsewhere, obscurely sensed something of genuine importance.

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Abbreviations

Ak. Kant., I. 1900–. *Kant's gesammelte Schriften*, herausgegeben von der Königlich Preußischen Akademie der Wissenschaften. Berlin and Leipzig.

BBGA Bolzano, B. 1969–. *Bernard Bolzano-Gesamtausgabe*, ed. E. Winter, J. Berg, F. Kambartel, J. Loužil, B. van Rootselaar. Stuttgart-Bad Cannstatt.

WL Bolzano, B. 1837. *Wissenschaftslehre.* Sulzbach. New ed. in **BBGA**, Series 1, Vols. 11/1-14/3. Partial Eng. tr. in B. Bolzano, *Theory of Science*, R. George ed. and tr. (Berkeley and Los Angeles, 1972).

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